COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – Spring 2008

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- All questions in this exam have a common theme: ***Sustainable Energy Systems***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1: Geometric Modeling

In this problem you will model part of the airfoil of a power generation wind turbine.

Assume that the section of the blade shown below can be modeled as a 3x2 degree Bezier surface patch, where the surface is cubic in the u direction and quadratic in the w direction. Control vertices $\mathbf{p}_{i,0}$ are labeled on the graph for guidance.

The control vertices of this patch are:

	p_{i0}	p_{i1}	p_{i2}
p_{0j}	0,0,0	0,50,0	0,100,0
p_{1j}	0,0,30	0,50,30	0,100,30
p_{2j}	20,0,10	20,50,18	20,100,20
p_{3j}	40,0,0	45,50,13	50,100,20

a) Derive the equation of the Bezier patch – expand the equation for $p(u,w)$ below so that your answer is in terms of the blending functions polynomials in u and w.

b) Now, derive the matrix form of the surface equations.

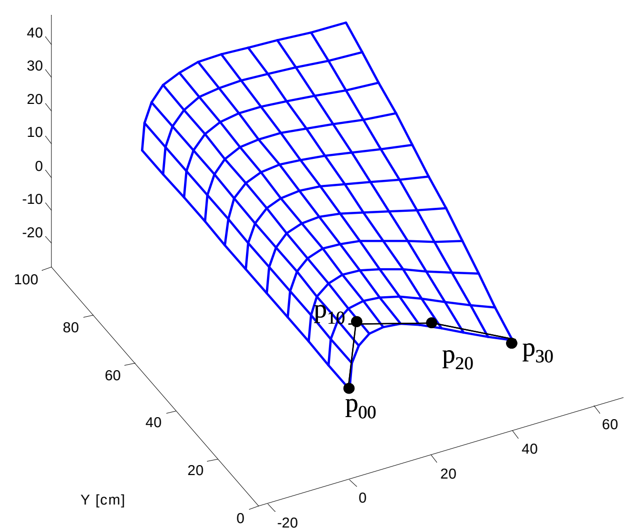
c) Compute the point on the surface at $u = 0.5$, $w = 1$.

d) Assume that G1 continuity is desired between surface patches. Compute the control vertex coordinates for the first two rows of CVs in order to achieve G1 continuity. Assume that the first row of CVs is at y = 0 (connect new patch along the curve with labeled CVs) and the second row is at y = -50.

Bezier surface equations:

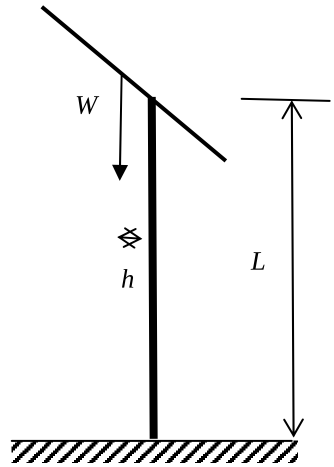
$$p(u,w) = \sum_{i=0}^{n}\sum_{j=0}^{m} \underset{\sim}{p}_{ij} B_{i,n}(u) B_{j,m}(w)$$

$$B_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i}$$



Question 2: Finite Element Analysis

An engineering firm is performing an anlysis of solar panels mounted on vertical posts, as shown in the figures. In their analysis, the firm found that certain panels were not mounted at their centroid, as illustrated in the schematic.



As part of the engineering firm, you are asked to determine the post tip displacement. Assume that the vertical support post has a height of L and is cylindrical with a diameter of D. Assume that the solar panel of weight W is mounted such that the horizontal distance of the solar panel centroid is h from the cylindrical post, as illustrated. Assume that the solar panel is currently positioned at an angle of 45° from the vertical post.

1. Use finite-element formulation to solve for the displacement at the post tip.
2. State all of your assumptions clearly.
3. Show the boundary conditions and loading conditions.
4. Starting with the element matrices, write down the assembly stiffness matrix.
5. Show all steps to determine the post tip displacement.

<u>Element A - Stiffness Matrix</u>

$$[K] = \frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} \qquad l = \frac{x_2 - x_1}{L} \qquad m = \frac{y_2 - y_1}{L}$$

where *E, A,* and *L* are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; *l* and *m* are direction cosines of the element with respect to *X* and *Y* axes.

<u>Element B - Stiffness Matrix</u>

$$[K] = \frac{2EI}{L^3}\begin{bmatrix} 6 & -3L & -6 & -3L \\ -3L & 2L^2 & 3L & L^2 \\ -6 & 3L & 6 & 3L \\ -3L & L^2 & 3L & 2L^2 \end{bmatrix}$$

where E, I, and L are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

Problem 3: Numerical Methods

In an effort to make your own contribution to combating global warming, you have decided to replace the internal combustion engine in your car with an electric motor. Since you have a small car, you are confident that you can find an electric motor that provides sufficient power and torque to maintain a safe and enjoyable ride. To help you select an appropriate motor, you need to perform a simulation that predicts the acceleration potential of your electric car, expressed as the 0-100km/h time. There are two steps in this process: 1) defining a model, 2) solving the model.

The model is given by the following equations:

$$V = Ri + L\frac{di}{dt} + K\omega \tag{1}$$

$$\tau_{motor} = Ki - B\omega - J\frac{d\omega}{dt} \tag{2}$$

$$\tau_{load} = n\left(0.5(n\omega)^2 + mn\frac{d\omega}{dt}\right) \tag{3}$$

with $V = 120\text{V}$, $R = 0.05\Omega$, $L = 0.01\text{H}$, $K = 0.3\text{Nm/A}$, $B = 0.3\text{Nms/rad}$, $J = 0.2\text{kgm}^2$, $n = 0.06\text{m/rad}$, $m = 1500\text{kg}$ (all quantities in SI units).

Equation (1) represents the electrical behavior of the motor, Equation (2) represents the mechanical behavior, and Equation (3) represents the load experienced by the motor due to aerodynamic drag and the inertia of the car body. The torque produced by the motor is used entirely to overcome the load torque.

Task 1: Write this model as an explicit set of ordinary differential equations of the form $\dot{y} = f(t, y)$, where y is the state vector.

Task 2: Using Heun's method with a time step of 0.2 seconds, determine the speed of the motor at 0.4 seconds. Assume that at time zero the car is standing still and the driver has just switched on the power, connecting the battery to the motor. Heun's method is defined as:

$$\begin{cases} y_{i+1} = y_i + h\phi \\ \phi = (k_1 + k_2)/2 \\ k_1 = f(t_i, y_i) \\ k_2 = f(t_i + h, y_i + hk_1) \end{cases}$$

(Note: If you were unable to solve Task 1, then show your knowledge of Heun's method using a second order differential equation of your choice — you will receive partial credit)

Question 1: Heun's method is a Runge Kutta method. If it were your choice, would you choose a different Runge Kutta method to solve this problem? Justify your answer.

Question 2: The numerical error of your solution in Task 2 is approximately 3 rad/s — that is too large. Assume you would iterate until 0.4 seconds but with a time step of 0.01 rather than 0.2. What would be a good estimate of the numerical error? Justify you answer.

Question 3: To predict the 0-100km/h time of the car, you would have to simulate for about 20 seconds rather than just 0.4 seconds. Assume that in this case you would use a time step of 2 seconds. Comment on the accuracy of your simulation in this scenario.