GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Quarter 1997

Computer-Aided Design
EXAM AREA

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Assigned Number (DO NOT SIGN YOUR NAME)

- Please sign your name on the back of this page—

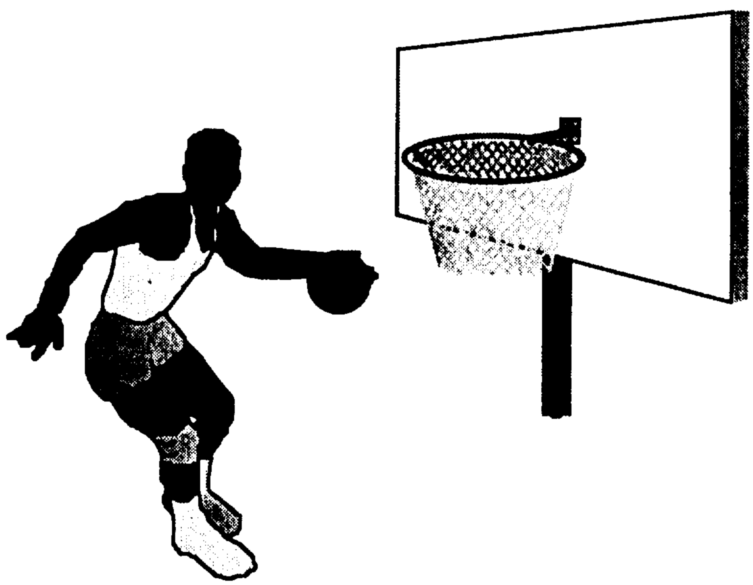
GEORGIA INSTITUTE OF TECHNOLOGY
GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENGINEERING

COMPUTER-AIDED DESIGN
PH.D. QUALIFYING EXAM

Spring 1997

Bras, Fulton, Rosen (Chair)

Yes it is that time of year again - the National Basketball Association (NBA) playoffs have started!! The slam dunks, players diving for balls, 3-point shots from almost mid-court ... it will be exciting! And the equipment must be very sturdy. No one likes waiting around for a shattered backboard to be replaced or a collapsed basket support to be fixed. For this exam we are asking you to tackle three problems associated with the modeling and analysis of the basketball backboards and supports:



We are interested in learning what you know and your ability to reason. If for some reason you do not follow the question or are confused, kindly adjust the question suitably and proceed with your answer. Please structure your answers as follows:

1) Restate the problem in your own words, identifying any assumptions, judgments, and adjustments that you are making.
2) Tell us your strategy or plan for solving this problem.
3) Solve the problem
4) Tell us about any insight you gained by solving this problem.

Oral Exam Note

When you come to the oral exam, be prepared to comment briefly on your research activities and where CAE/CAD technology fits into that research.

Problem 1

As you know, basketball players are notorious for making slam dunks. These dunks introduce a high force on the basket rim. A company is interested in marketing new composite basketball rims, but wants to know whether these new rims can withstand Michael Jordan's slam-dunks.

A student has found an empirical formula (by means of curve fitting) for estimating the energy introduced in the rim through an average Michael Jordan slam dunk. The formula

$$E(x) = 0.2 + 25\,x - 200\,x^2 + 675x^3 - 900\,x^4 + 400\,x^5$$

measures the energy at a specific location x on the circumference of the rim. x is measured in meters. Assume that x = 0 is where the rim attaches to the backboard. Assume that the total circumference of the rim is 0.8 m.

The company is interested in finding the total energy contained in the rim after a slam-dunk.

a) Perform an integration of E(x) using the Romberg integration algorithm.

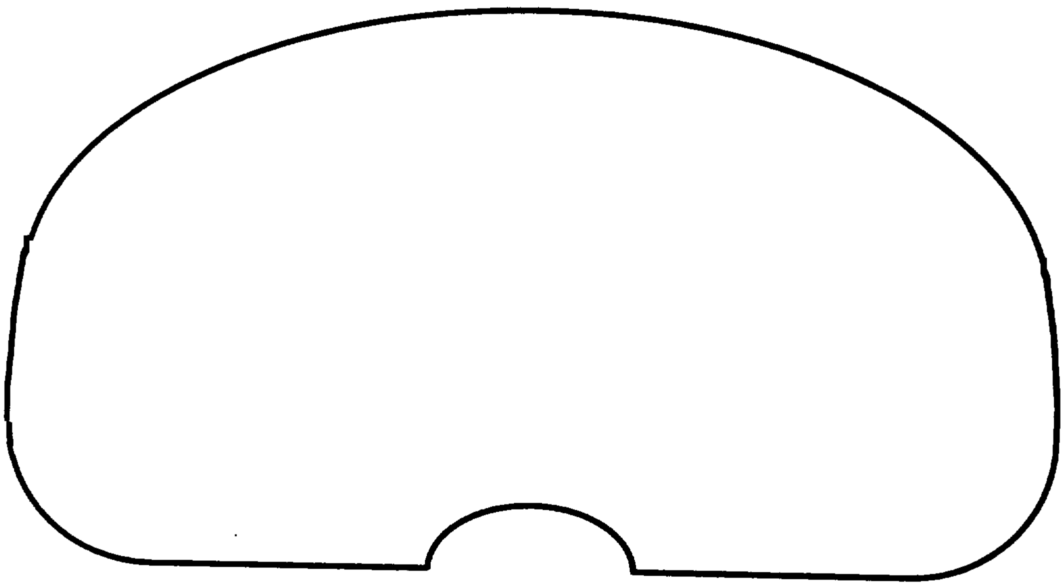
b) Perform the same integration using the trapezoidal rule and comment on the difference of the results obtained by the Romberg and trapezoidal solution methods.

c) Explain what Richardson's extrapolation is and how it relates to the preceding two questions.

d) Name and explain some other methods to solve the above integration and discuss their pros and cons.

Problem 2

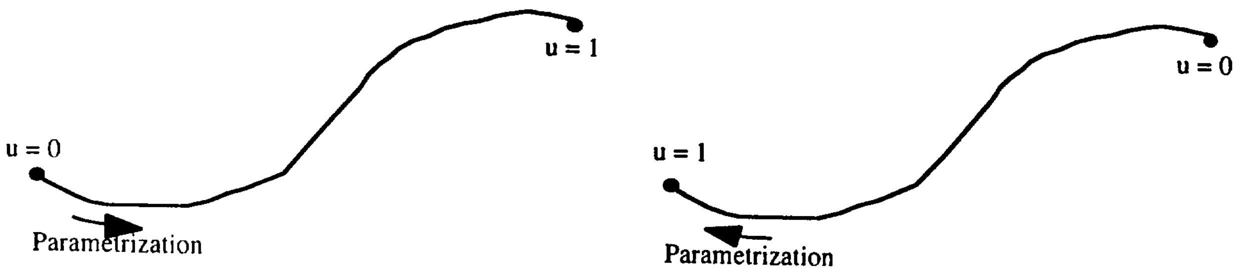
In the figure below, the outline of the basketball backboard is shown. This problem is about modeling shapes geometrically and using appropriate geometric properties.



a Indicate how you would use composite, cubic Bezier curves to model the indicated backboard shape. Show control vertices and control polygons. Label points where curves join.

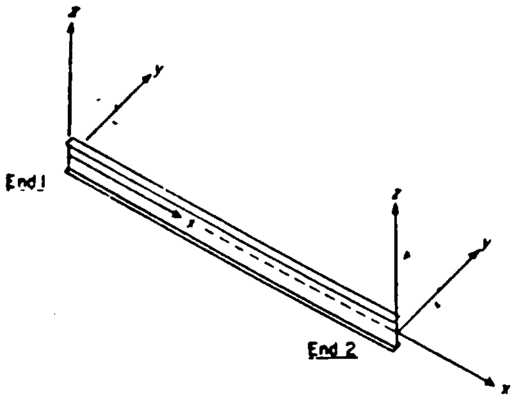
b Describe how you modeled continuity between curve segments. Also describe how you utilized symmetry in your geometric model.

c <u>Prove that cubic Bezier curves are symmetric</u>. That is, the direction of curve parametrization does not matter. See the diagram below. On the left, the curve is parameterized from left-to-right, while the curve on the right is parametrized in the opposite direction. Why is this possible? Are other orders of curves also symmetric? What about surfaces?



Problem 3

The Shaq Attack! You are given the finite element, three dmiensional force displacement relationships for the beam shown below:



Member coordinate system for a member of a three-dimensional frame.

$$\left.\begin{array}{l} \boldsymbol{p}_1 = \boldsymbol{K}_{11}\boldsymbol{d}_1 + \boldsymbol{K}_{12}\boldsymbol{d}_2 \\ \boldsymbol{p}_2 = \boldsymbol{K}_{21}\boldsymbol{d}_1 + \boldsymbol{K}_{22}\boldsymbol{d}_2 \end{array}\right\}$$

where

$$\boldsymbol{p} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ m_x \\ m_y \\ m_z \end{bmatrix} \qquad \boldsymbol{d} = \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix}$$

and

$$\boldsymbol{K}_{11} = \begin{bmatrix} EA/L & 0 & 0 & 0 & 0 & 0 \\ 0 & 12EI_z/L^3 & 0 & 0 & 0 & 6EI_z/L^2 \\ 0 & 0 & 12EI_y/L^3 & 0 & -6EI_y/L^2 & 0 \\ 0 & 0 & 0 & GJ/L & 0 & 0 \\ 0 & 0 & -6EI_y/L^2 & 0 & 4EI_y/L & 0 \\ 0 & 6EI_z/L^2 & 0 & 0 & 0 & 4EI_z/L \end{bmatrix}$$

$$\boldsymbol{K}_{12} = \boldsymbol{K}'_{21} = \begin{bmatrix} -EA/L & 0 & 0 & 0 & 0 & 0 \\ 0 & -12EI_z/L^3 & 0 & 0 & 0 & 6EI_z/L^2 \\ 0 & 0 & -12EI_y/L^3 & 0 & -6EI_y/L^2 & 0 \\ 0 & 0 & 0 & -GJ/L & 0 & 0 \\ 0 & 0 & 6EI_y/L^2 & 0 & 2EI_y/L & 0 \\ 0 & -6EI_z/L^2 & 0 & 0 & 0 & 2EI_z/L \end{bmatrix}$$

and $\boldsymbol{K}_{22}$ is equal to $\boldsymbol{K}_{11}$ with the signs of the off-diagonal elements $-6EI_y/L^2$ and $6EI_z/L^2$ reversed.

Use these properties to estimate the deflection at the end of a hoop caused by a 300 pound person hanging from a very stiff hoop supported by a flexible post. The post has a square cross-section and is made of plastic with a Youngs modulus of 100,000 psi.
Is this a good design? Explain your answer.