COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – FALL 2010

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- All questions in this exam have a common theme: ***Chilean Mine Rescue***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Problem 1 - Geometric Modeling

The authorities digging the tunnels to rescue the trapped Chilean miners were debating whether or not to reinforce the first tunnel with pipe. This decision was complicated since the tunnel might not be straight; rather it might be curved. How long a piece of pipe could be inserted into the tunnel without being restricted by the curves?

A schematic of the top part of the tunnel is shown below. Also shown is a geometric model that will be used to investigate the issue of maximum pipe length. Assume the centerline of the tunnel is modeled using a Bezier curve with control vertices (0,0), (20, -800), (300, -300), (400, -1000). If the tunnel is modeled having a radius of 0, then the equivalent pipe radius, r_e, should be the clearance distance between the tunnel and pipe (i.e., $r_e = r_t - r_p$), where r_t is the tunnel radius and r_p is the original pipe radius. If the tunnel centerline remains within the equivalent pipe, then the actual pipe will not interfere with the tunnel walls.

Assume the pipe is 24 inches in diameter (outside) and the tunnel is 36 inches in diameter, so that $r_e = 6$ inches. Answer the following questions:

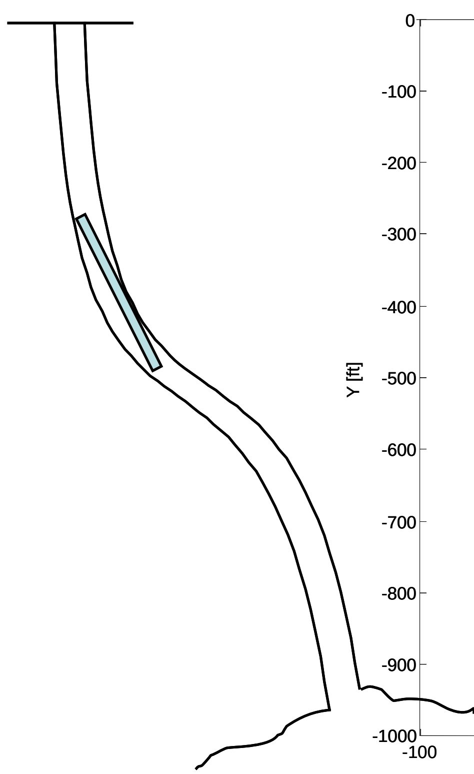
a) If one end of the pipe is at u = 0.3 and the other end is at u = 0.5, compute the x,y coordinates of the end points.

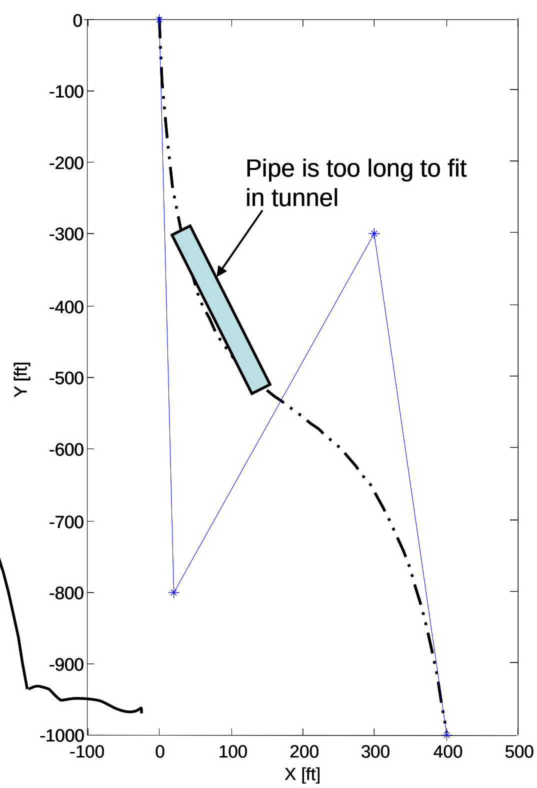
b) How long is the pipe?

c) Estimate the clearance between the pipe and tunnel: compute several points on the curve and compute the distance from those points and the pipe centerline. Based on your estimate, does the pipe fit? Explain. (the distance between a point and a line is: $d = \frac{\left|\overrightarrow{p_1 p_2} \times \overrightarrow{p_1 q}\right|}{\left|\overrightarrow{p_1 p_2}\right|}$, where p_1 and p_2 are points on the line and q is a point on the tunnel centerline.)

d) Assume the pipe end point at u = 0.3 is fixed. Assume the other end point is a variable. Explain how you would determine the maximum pipe length that will fit in the tunnel at this position.

e) Explain how you would solve the more general problem, that is, what is the longest pipe that will fit down the tunnel. We are looking for explanations that involve geometric models, geometric reasoning, and numerical methods.





Problem 2 – Finite-Element Analysis

In the initial design phase, one team designed a bracket as shown Figure 2.1 to support the pulley that will be used to hoist the trapped miners. Assume that the thickness of the bracket is of similar magnitude as the planar dimensions of the bracket. Assume that the bracket is rigidly attached on the top to the supporting frame and that the net force acting on the bracket is P. You are asked to determine the stress distribution in the bracket using finite-element analysis.

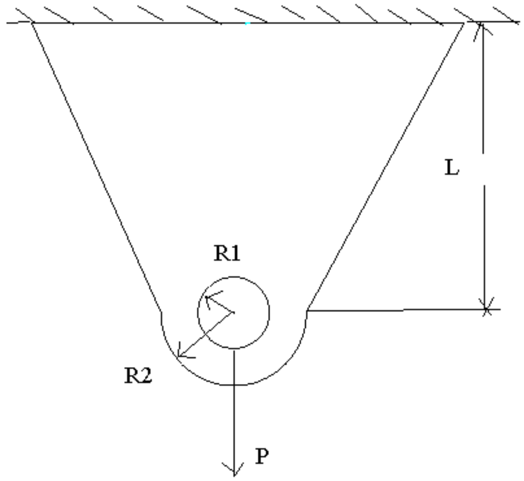


Figure 2.1 Bracket Design 1

1. Using symmetry wherever possible, show a representative finite-element mesh for the following cases with loading and boundary conditions.

    a. Material is isotropic.
    b. Material is orthotropic.

2. Figure 2.2 shows a modified horizontal positioning of the bracket which is rigidly attached on the left side. Assume that the material is isotropic. Show a representative finite-element mesh with loading and boundary conditions to perform the stress analysis.

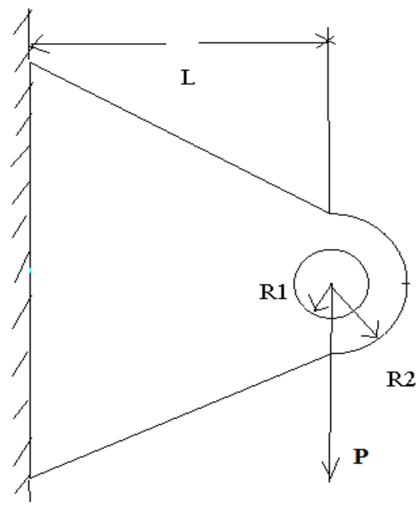
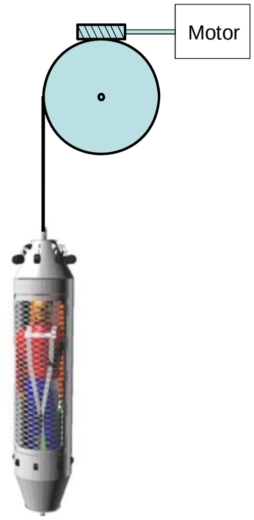


Figure 2.2 Bracket Design 2

For all of your meshes, ensure that

a. the elements are appropriately interconnected
b. the mesh density is able to capture the stress distribution in critical regions
c. the chosen element type(s) will be appropriate for the analysis. Discuss various element type(s) that you will choose for the analysis and justify.

Problem 3 – Numerical Methods



After being trapped at 620m under the Earth's surface for several days, the miners in Chile were finally freed using a rescue pod. While planning the rescue, you were asked to determine how long it would take to haul each miner to the surface. An image of the electro-mechanical winch mechanism is shown on the right. The motor characteristic of the AC squirrel cage motor is shown below. For rotational speeds between 100 and 160 rad/s, the motor characteristic can be well approximated using the following polynomial:

$\tau = a_0 + a_1\omega + a_2\omega^2 + a_3\omega^3 + a_4\omega^4$ with the torque, τ, expressed in Nm, and the rotational velocity, ω, expressed in rad/s. The coefficients are:

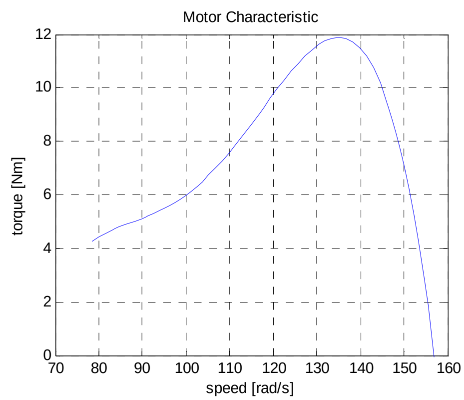
$$a_0 = -3\times10^2\ Nm$$

$$a_1 = 1.233\times10^1\ Nm/(rad/s)$$

$$a_2 = -1.866\times10^{-1}\ Nm/(rad/s)^2$$

$$a_3 = 1.233\times10^{-3}\ Nm/(rad/s)^3$$

$$a_4 = -3\times10^{-6}\ Nm/(rad/s)^4$$



The motor drives the winch drum through a worm drive with a 100:1 gear ratio. The drum radius is 0.3m. The rescue pod has a mass of 250kg and the miner is assumed to be 80kg. In addition to the gravitational force, the winch has to overcome friction and drag that can be modeled as proportional to the translational velocity of the pod with a constant of proportionality equal to 500 N/(m/s).

Questions:

a) Compute how long it takes to haul each miner the 620m to the surface. You can assume that the pod achieves its steady state velocity instantaneously. Find the solution with an accuracy of 1 rad/s. Show your work.

b) Justify your choice of numerical method. Which other numerical methods could you have used? Which method is best suited for solving this problem?