COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – Fall 2008

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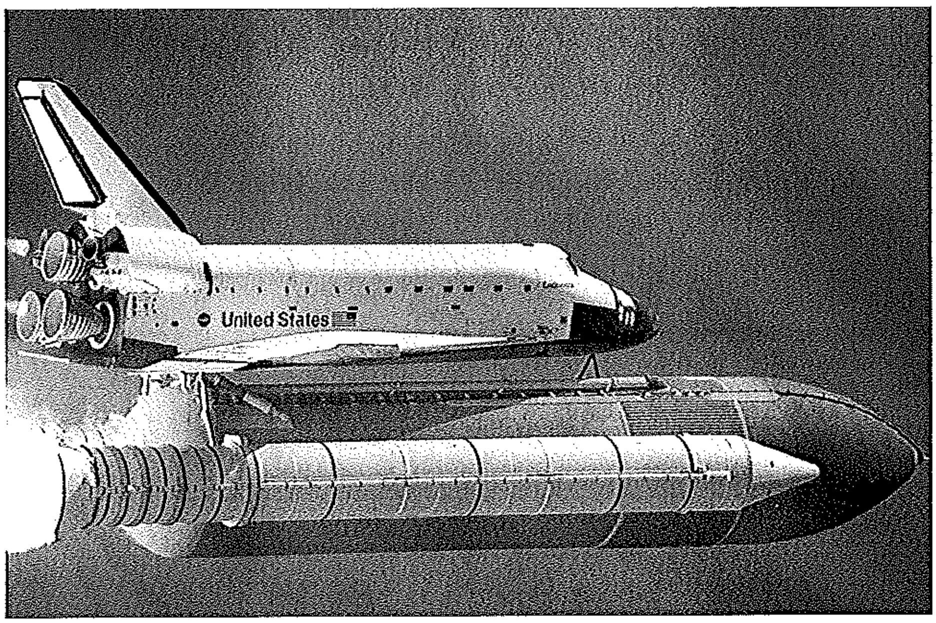
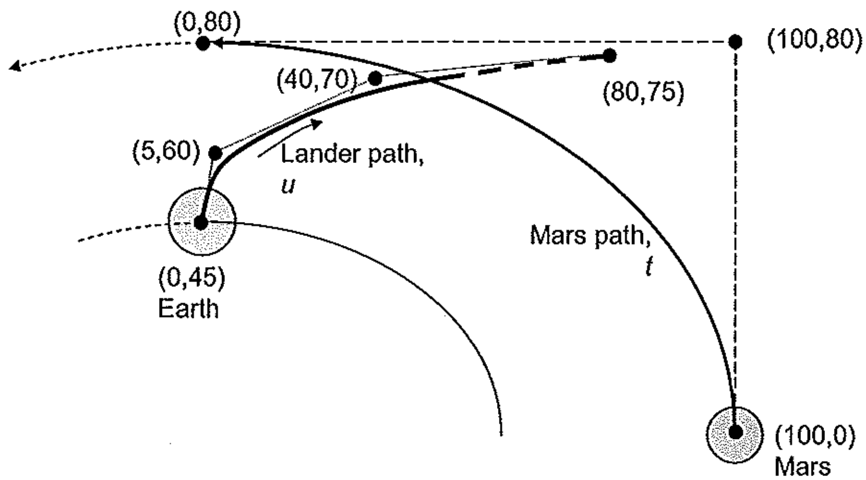


- All questions in this exam have a common theme: ***Space Systems***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1: Geometric Modeling

Let's say that the US is launching a spacecraft that is to land on Mars (call it a "lander"). Assume that the path taken by the lander can be approximated by a cubic Bezier curve, $\mathbf{q}(u)$. While the lander is zooming through space, Mars is also moving – it is revolving around the sun. Assume Mars' path can be approximated by a quadratic Bezier curve, $\mathbf{p}(t)$. For simplicity, assume both $\mathbf{q}(u)$ and $\mathbf{p}(t)$ are in the XY plane, that is the Z coordinate can be ignored. The situation is illustrated in the figure below, where coordinates for the control vertices are given. We are interested in having the lander land on Mars, meaning that the lander's path must intersect Mars' path. Find the conditions for their paths to intersect.



As a reminder, the equation for a cubic Bezier curve is:

$$\mathbf{q}(u) = \sum_{i=0}^{3} \mathbf{q}_i B_{i,3}(u)$$

where the $B_{i,3}$(t), etc., are the Bezier blending functions.

a. Describe the condition mathematically for the paths of the lander and Mars to intersect. Describe this condition verbally as well.

b. Derive the equations for the paths to intersect. Do not substitute control vertex values into the Bezier curve equations. I expect to see equations of the form:

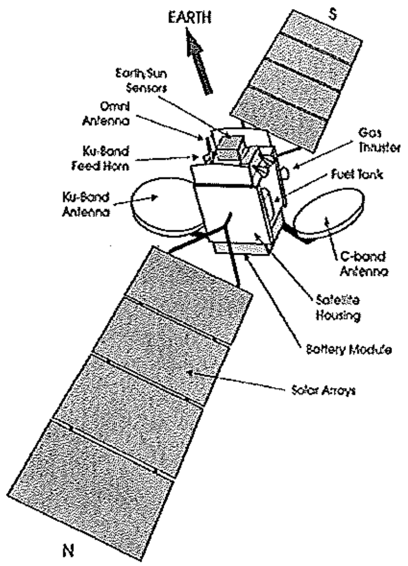
$$f_1(u,t) = 0$$
$$f_2(u,t) = 0$$

c. Identify a numerical method that can solve your equations. Explain the next several steps in applying your numerical method to solve the equations. Do not actually try to solve the equations as that will be time-consuming.

d. Assume that intersection occurs when the lander is at u = 0.7. Compute the X, Y coordinates of the lander (and Mars) at this intersection point.

Question 2: Finite Element Analysis

In satellite systems, the parts of solar arrays are often supported by cables and frame structures. As part of an engineering firm, you are asked to determine the deflections of the square frame. In the early stages of the design, you are considering simplified models of the frame structure as shown in Figure 2.



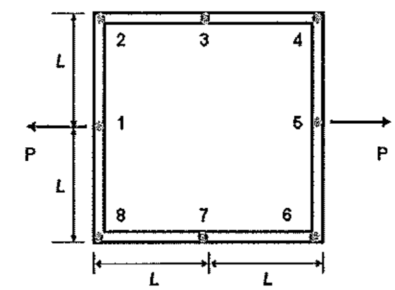


Figure 2. Square Frame for Solar Arrays

Figure 1. Satellite System

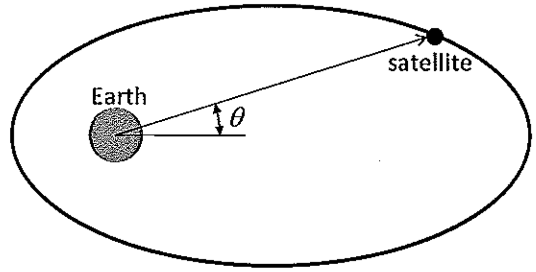
A square frame subjected to a pair of forces P at the nodes 1 and 5 is shown in Figure 2. It is assumed that the frame members are inextensible and that the right angles at the joints are preserved.

(i) Use the finite-element formulation to find the deflection shape.
(ii) State all of your assumptions clearly.
(iii) Show the boundary conditions and loading conditions.
(iv) Starting with the element matrices, write down the assembly stiffness matrix.
(v) Show all steps to determine the deflection.
(Note: the stiffness matrix of the frame element is given in Equation 1).

$$[K]=\frac{2EI}{L^3}\begin{bmatrix} 6 & 3L & -6 & 3L \\ 3L & 2L^2 & -3L & L^2 \\ -6 & -3L & 6 & -3L \\ 3L & L^2 & -3L & 2L^2 \end{bmatrix} \qquad \text{Eq.(1)}$$

Problem 3: Numerical Methods

Something went wrong during the launch of a satellite and it ended up in a very eccentric elliptical orbit around Earth, as is shown in the figure on the right. To re-establish communication with the satellite, you need to predict its angular position relative to Earth, $\theta(t)$.



The equations describing the orbit of the satellite are:

$$E(t) - e \cdot \sin(E(t)) = M_0 + n \cdot t \quad (1)$$

$$\theta(t) = \pi + \arccos\left(\frac{\cos(E(t)) - e}{1 - e \cdot \cos(E(t))}\right) \quad (2)$$

with $M_0 = 2$, $n = 0.001$ rad/s, and $e = 0.7$. (In case it helps you, E is called the eccentric anomaly, and $M_0 + n \cdot t$ is called the mean anomaly, but note that you do not need to understand orbital dynamics in order to solve this mathematical problem).

In your computation of $\theta(t)$, you can assume that your calculator has all the trigonometric functions built in.

3.1) From the perspective of numerical methods, what type of problem is this?

3.2) Name two numerical methods that can be used to solve this type of problem.

3.3) Discuss the advantages and disadvantages of these two methods for solving this specific problem, assuming that you may have to compute $\theta(t)$ for a variety of different values of t. Which method would you recommend using?

3.4) Now, solve the problem using the method of your choice. Describe the algorithm that you would use to determine $\theta(t)$ for a given value of t. Then, apply the algorithm to solve for $\theta(t)$ at $t = 10^4$ seconds. Find the solution with a relative accuracy of 0.001. Show your work, including the verification of the required accuracy.