COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – Fall 2006

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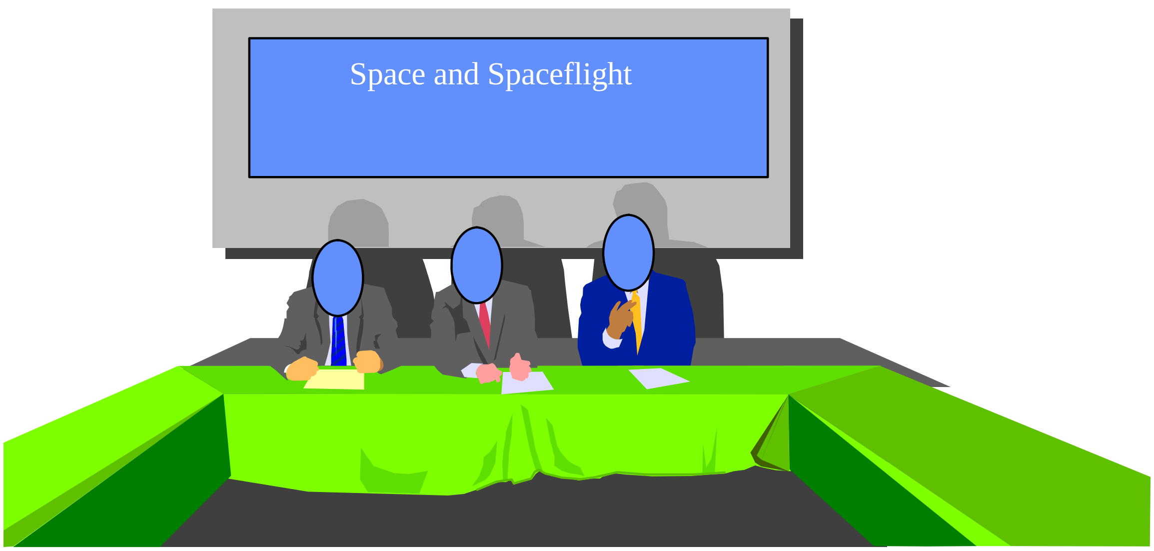


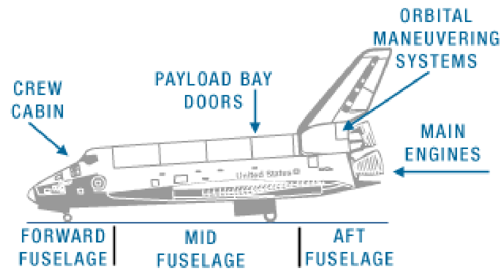
- All questions in this exam have a common theme: ***Space and Spaceflight***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1 - Geometric Modeling

In this problem you will model the front-end of a space shuttle geometrically. Consider the current space shuttle shown to the right. The specific curvatures of the front-end reflect a balance among aerodynamics, strength, cockpit space, and manufacturability. I am only interested in the front-end shape (forward fuselage and crew cabin areas, not the payload bay door area or mid-fuselage).

Develop a new shape for the forward fuselage.



a) Sketch a 2D curve representing the front fuselage shape (profile) on your answer sheet. Develop a model of the curve as a 2D composite, cubic Bezier curve. Ensure G1 continuity between curve segments. Explain how you ensure G1 continuity. Show the curves with control vertices.

b) Sketch the same 2D curve again. This time model the curve as a single cubic B-spline curve. Explain why this is possible (single curve segment) with B-splines, but not with Bezier curves.

c) Will your composite cubic Bezier curve have exactly the same shape as your single cubic B-spline curve? Why or why not? Explain.

d) Assume that a cubic Bezier curve has the following control vertices: (20,5), (1,6), (22,-1), (0,0). Compute the 2D (X, Y) coordinates of the point at $u = 0.7$ on this curve.

e) Derive the equations for a linear B-spline curve from point (1, 0) to point (11, 5). Show your derivation and explain your approach, including any short-cuts you take.

Bezier curve equations:

$$b(u) = \sum_{i=0}^{n} \underset{\sim}{p}_i B_{i,n}(u)$$

$$B_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i}$$

B-spline curve equations:

$$\mathbf{p}(u) = \sum_{i=0}^{n} \mathbf{p}_i N_{i,k}(u)$$

$$N_{i,1}(u) = \begin{cases} 1, \; if \;\; t_i \le u < t_{i+1} \\ 0, \; otherwise \end{cases}$$

$$N_{i,k}(u) = \frac{(u - t_i) N_{i,k-1}(u)}{t_{i+k-1} - t_i} + \frac{(t_{i+k} - u) N_{i+1,k-1}(u)}{t_{i+k} - t_{i+1}}$$

Question 2 – Finite Element Analysis

A triangular structure in the spacecraft has a base width of 2 m and a length of 4 m. The thickness of the structure is 10 mm. The modulus of elasticity of the structure is 120 GPa. The base of the triangle is rigidly attached to a surface as shown in the figure below. The apex is of the triangle is attached to another rigid surface with a string of 2 mm length. The string is capable of offering resistance only under tension. A horizontal sinusoidal load $P\ (t) = 7500 \sin \pi t$ kN is applied at a distance of 2 m from the base as shown in the figure.

You are asked to determine the displacement at the point where the load is applied, when time t varies from 0 to 1 s.

Use an appropriate finite-element formulation to solve the problem.

1. State all of your assumptions clearly.
2. Show all of your calculations.
3. Show the boundary conditions and loading conditions.
4. Write down element stiffness matrix and assembly stiffness matrix.
5. Determine how the displacement at the point where the load is applied will change, when the time t varies from 0 to 1 s.

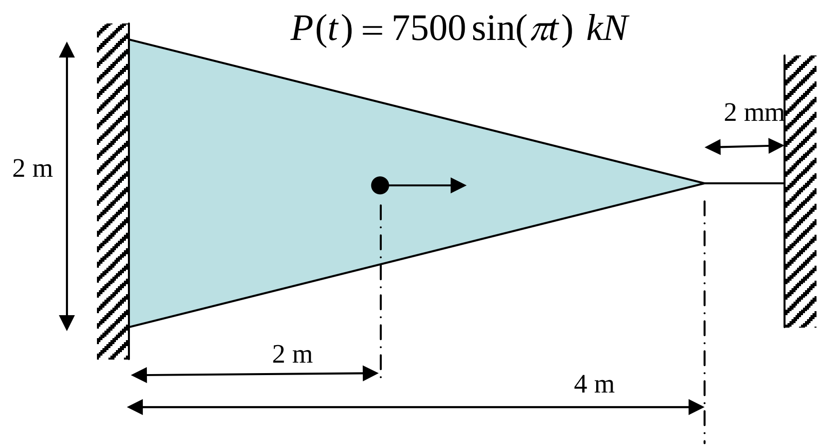


Figure not to

Element A - Stiffness Matrix

$$[K] = \frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} \qquad l = \frac{x_2 - x_1}{L} \qquad m = \frac{y_2 - y_1}{L}$$

$$[K] = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

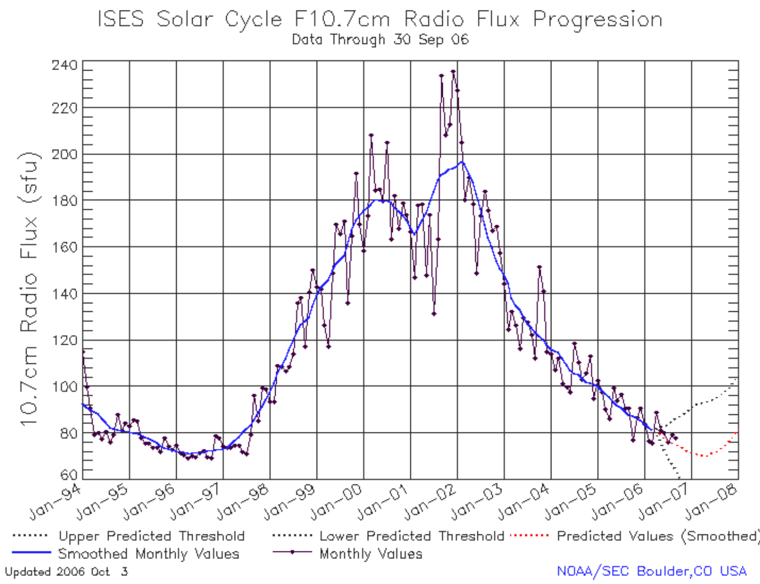
where E, A, and L are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; l and m are direction cosines of the element with respect to X and Y axes.

Element B - Stiffness Matrix

$$[K] = \frac{2EI}{h^3}\begin{bmatrix} 6 & -3h & -6 & -3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix}$$

where E, I, and h are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

Question 3 – Numerical Methods



The dynamics of Low-Earth Orbit satellites are significantly influenced by the aerodynamic drag. Although the air density is very small at these altitudes, the drag will still slow down the satellite sufficiently that it will lose altitude and burn up in the atmosphere unless it is provided a small boost on a regular basis to regain its original velocity. In designing a satellite, it is therefore important to have a good model for the air density. But modeling the air density turns out to be challenging because it depends strongly on the solar cycle. Increases in solar activity make the atmosphere swell up and reach further into space, increasing the drag on satellites significantly. A typical satellite flying at an initial altitude of 500km would last about 30 years under the conditions of a solar cycle minimum while only 3 years under a solar cycle maximum.

You are in charge of developing a model for the solar activity. Given the historical data below, you decide as a first approximation to fit a polynomial to the data and then use the best fit polynomial in your orbit calculations for the satellite design you are working on.

Time: t_i [years since Jan 1 2005]	0	0.25	0.5	0.75	1	1.25	1.5	1.75
Flux: y_i [sfu]	104	85	96	76	84	89	76	82

$$\sum_{i=1}^{8} t_i = 7 \quad \sum_{i=1}^{8} t_i^2 = 8.75 \quad \sum_{i=1}^{8} t_i^3 = 12.25 \quad \sum_{i=1}^{8} t_i^4 \cong 18.2656 \quad \sum_{i=1}^{8} t_i^5 \cong 28.3281 \quad \sum_{i=1}^{8} t_i^6 \cong 45.1221$$

$$\sum_{i=1}^{8} y_i = 692 \quad \sum_{i=1}^{8} t_i y_i = 579 \quad \sum_{i=1}^{8} t_i^2 y_i = 717.25 \quad \sum_{i=1}^{8} t_i^3 y_i \cong 999.19$$

$$\sum_{i=1}^{8} t_i y_i^2 = 48134.5 \quad \sum_{i=1}^{8} t_i y_i^3 = 4022404.5$$

Questions:

a. Determine the coefficients of a parabola that fits the data above in a least-squares sense. Start by deriving the equations for determining the linear least-squares fit of a parabola.
b. Which metric would you use to assess the goodness of fit of your solution?
c. To improve the accuracy of your model, you decide to consider higher-order polynomials. What are some of the considerations in determining the best ***order*** of the polynomial for your model?
d. Provide your assessment of the overall approach suggested in this problem: Is using a polynomial curve fit a good method for modeling future solar activity?