COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – Fall 2004

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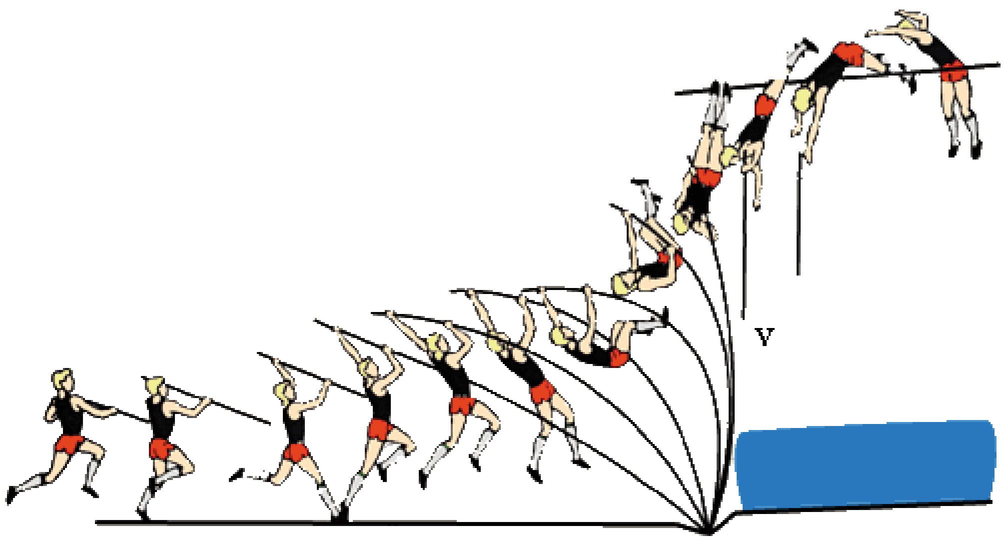

- All questions in this exam have a common theme: ***Pole Vaulting***

- Answer **all** questions.

- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.

- Show all steps and calculations.

- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1
Finite Element Modeling

A 5m long pole with a hollow circular cross-section having an inner diameter of 0.028m and an outer diameter of 0.030m is used in a pole vault competition. The pole is made of a composite material with a modulus of elasticity of 35 GPa. An 80-kg athlete is using such a pole for pole vaulting. An engineer estimates that a horizontal force of 1000 N may be exerted, when the pole is vertical during the jump. The engineer also assumes that the center of gravity of the athlete is at the tip of the pole.

Using appropriate finite elements, determine the deflection of the tip of the pole when the pole is vertical. In your calculations, assume that the bottom of the pole is fixed to the ground. State all your assumptions clearly.

Element 1 - Stiffness Matrix

$$[K]=\frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

$$l=\frac{x_2-x_1}{L}$$

$$m=\frac{y_2-y_1}{L}$$

where *E*, *A*, and *L* are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; *l* and *m* are direction cosines of the element with respect to *X* and *Y* axes.

$$[K]=\frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Element 2 - Stiffness Matrix

$$[K]=\frac{2EI}{h^3}\begin{bmatrix} 6 & -3h & -6 & -3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix}$$

where *E, I,* and *h* are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

Question 2
Numerical Methods

A new track and field event is proposed – catapulting - in which the pole vault pole is used to jump for maximum horizontal distance. A schematic of the event is presented in Figure 2, which also indicates the analytical variables used in calculating the total distance jumped. This distance, denoted by X_T, is equal to the sum of $X_1+X_2+X_3$ and can be calculated by solving the equations of motion of an athlete of mass *m* influenced by gravity *g* and air resistance. Note that the transition from X_2 to X_3 is where the athlete releases the pole and is traveling with velocity v_2.

$$\begin{cases} a_x = \dfrac{dv_x}{dt} = -\dfrac{1}{m} F_D \cos u \\ a_y = \dfrac{dv_y}{dt} = -\dfrac{1}{m}(F_D \sin u + mg) \qquad \text{where } v = \sqrt{v_x^2 + v_y^2} \text{ and } u = \tan^{-1}\left(\dfrac{v_y}{v_x}\right) \\ F_D = \dfrac{1}{2}\rho A C_D v^2 \qquad \text{(Drag force)} \end{cases}$$

The drag force *FD* acting on the projectile depends on air density ρ, velocity *v*, cross section area *A* and drag coefficient C_D.

Solve for the position and velocity of the system using Euler's method and the 4th order Runge-Kutta method to find X_T for the following given parameter values:

$L_p = 6$m	$v_1 = 10.0$ m/s	$\theta = 24^0$	$\alpha = 40^0$
m = 75kg	A = 0.5 m^2	$C_D = 0.9$	$v_2 = 10.0$ m/s
$\rho = 1.225$ kg/m^3 g = 9.8m/s^2			

Use time steps of 1s and compare results. Are they different? If so, why? How would you calculate maximum height achieved by the athlete after releasing the pole?

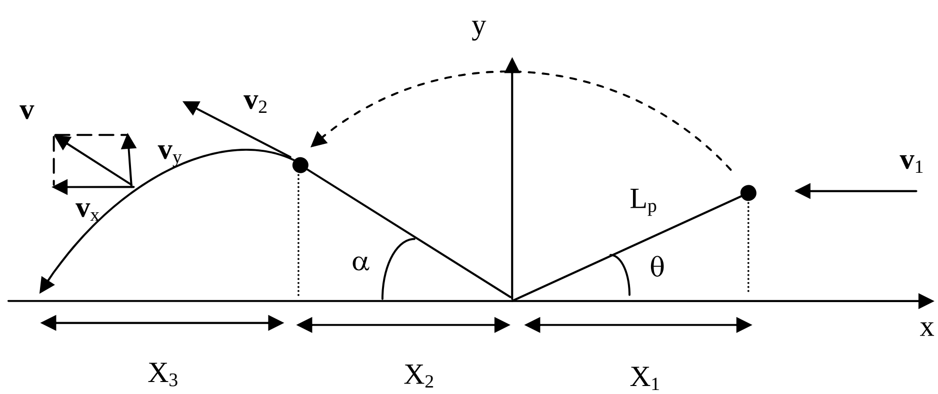

Figure 2 Schematic of catapulting, indicating analytical variables used in calculating X_T

Question 3

Geometric Modeling

When a world class pole vaulter vaults, he bends the pole considerably in order to catapult himself over the bar. I am interested in developing a geometric model of bending pole to support visualization, analysis, and simulation purposes. Hence, the model should be as accurate as possible.

$$b(u) = \sum_{i=0}^{n} B_{i,n}(u)\vec{P}$$

$$B_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i}$$

The equation for a Bezier curve is included at right.

Answer the following questions:

a. Assume that a pole vaulting pole is 4.5 m long. Sketch a bent pole, similar to the picture above, and show how you would model the pole as a cubic Bezier curve. Assign coordinates to the control vertices, stating any assumptions that you need to make. Derive the equation of the cubic Bezier curve that models the centerline of the pole using your control vertices.

b. Compute the coordinates of the point on the Bezier curve at u = 0.8.

c. Compare and contrast a cubic Hermite model of the pole with the Bezier formulation.

d. Compare and contrast a cubic b-spline model of the pole with the Bezier formulation.

e. Explain how you would derive an expression for the energy stored in a pole using your Bezier curve model. Could you use the same approach for the Hermite and b-spline curves?

f. Given your models and analysis, which curve formulation would you select to achieve the greatest accuracy?