GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam – Fall Semester 1999

COMPUTER AIDED ENGINEERING
Exam Area

Assigned Number (**DO NOT SIGN YOUR NAME**)

-- Please sign your name on the back of this page --

GEORGIA INSTITUTE OF TECHNOLOGY
GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENGINEERING

COMPUTER-AIDED DESIGN
PH.D. QUALIFYING EXAM
Fall 1999

Yes, it is football season again, and the Georgia Tech Yellow Jackets are nationally ranked! There are a number of issues that the football team needs assessed in order to make the season a complete success. The following problems cover some of these.

Joe Hamilton, GT Quarterback, runs for positive yardage.

We are interested in learning what you know and your ability to reason. If for some reason you do not follow the question or are confused, kindly adjust the question suitably and proceed with your answer. Please structure your answers as follows:

1) Restate the problem in your own words, identifying any assumptions, judgments, and adjustments that you are making.
2) Tell us your strategy or plan for solving this problem.
3) Solve the problem.
4) Tell us about any insight you gained by solving this problem.

Oral Exam Note

When you come to the oral exam be prepared to comment briefly on your research activities and where CAE/CAD technology fits into that research.

Question 1.

Field goals have been a problem for the Georgia Tech football team in the past. This year, they want a more high-tech approach to ensure that more field goals are made. They have derived a differential equation that models a football's flight when kicked, as a function of the football's initial height, velocity, acceleration, and jerk (derivative of acceleration).

For the purposes of simplicity, the governing differential equation has been simplified and initial conditions have been normalized. The football team is interested in the solution process so that they can apply the process to equations that represent particular kickers and situations.

The governing differential equation is given below:

$$\frac{d^3y(x)}{dx^3} - \frac{d^2y(x)}{dx^2} - 2\frac{dy(x)}{dx} - xy(x) = 0$$

Initial Conditions are:

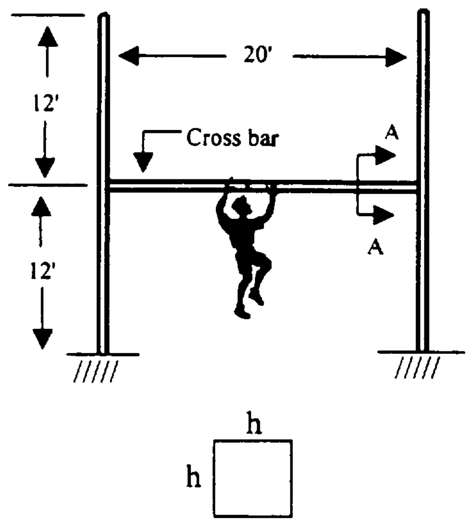
$$\left.\frac{d^2y(x)}{dx^2}\right|_{x=0} = \left.\frac{dy(x)}{dx}\right|_{x=0} = y(0) = 1$$

In this equation, x represents the horizontal axis and y represents the height of the football.

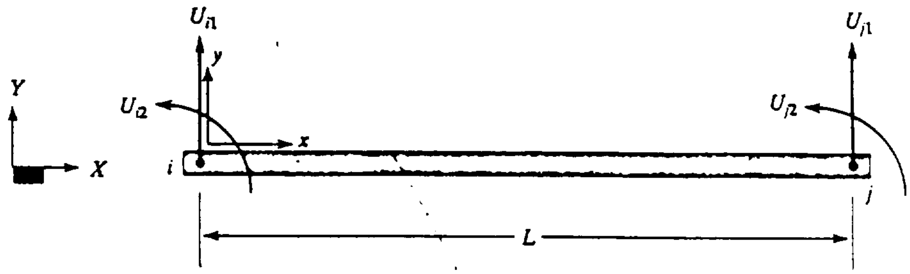
Find y(1.5) using a numerical method for solving ordinary differential equations. Write all the algorithm steps and intermediate results. The method to be used should not involve iteration step size greater than 0.5.

2a. A 300-pound Tech defense tackle intercepts the football and rumbles for a touchdown. In his excitement, he jumps up and hangs from the center of the goal post horizontal cross bar. Use a 2-element beam finite element approach to model the cross bar and provide a formula to estimate the resulting deflection at the center of the cross bar. Assume the cross bar is fixed at each end. The beam element stiffness matrix is shown below.

2b. If the cross bar has a solid square cross-section and is made of aluminum with Youngs modules E = 10×10^6 psi and yield strength σ = 50,000 psi, what would you chose as the member cross-section height h to support such linemen behavior.



Section A-A



A beam element.

$$\{P\}=[K]\{U\}=\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} U_{i1} \\ U_{i2} \\ U_{j1} \\ U_{j2} \end{Bmatrix}$$

Question 3.

The face-mask in front of the helmet is critical to the football player's safety. It prevents opposing players from hitting your face, keeps fingers out of your eyes, etc. You must specify the geometry of a face-mask for a football helmet. That is, specify the curves, control vertices, and continuity conditions among the bars on helmets that typically protect the face of a football player.

Given

Equations for **Bezier curves** are

$$b(u) = \sum_{j=0}^{n} B_{i,n}(u)\vec{P}$$

$$B_{i,n}(u) = \binom{n}{i} u^{i}(1-u)^{n-i}$$

where: **P** are the control vertices that define the Bezier curve.

Questions

a) Given the equation for a Bezier curve above, derive the equations for a quadratic Bezier curve and a cubic Bezier curve.

b) Select points in the plane that will serve as the control vertices for a quadratic Bezier curve, then specify the coordinates of those points. Sketch the quadratic Bezier curve that fits those points. Plug your coordinates into the quadratic Bezier curve equation from part (a). Compute the X-Y coordinates on the curve corresponding to u = 0.5.

c) Face-Mask Modeling: Sketch two views (front and side views, for example) of a face-mask for a football helmet. Sketch a pictorial 3-dimensional view also. Indicate on your sketches how you could model the face-mask using linear, quadratic, and cubic Bezier curves. Please make your sketches large enough that you can label the Bezier curves.

d) Continuity: Describe the continuity conditions among your various Bezier curves. Select two adjacent Bezier curves in your face-mask. Sketch them again and show their control vertices. Explain the continuity requirements at the interface between the curves and how you achieved that continuity requirement.