GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Quarter 1996

COMPUTER-AIDED ENGINEERING

EXAM AREA

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Assigned Number **(DO NOT SIGN YOUR NAME)**

-- **Please sign your name on the back of this page** --

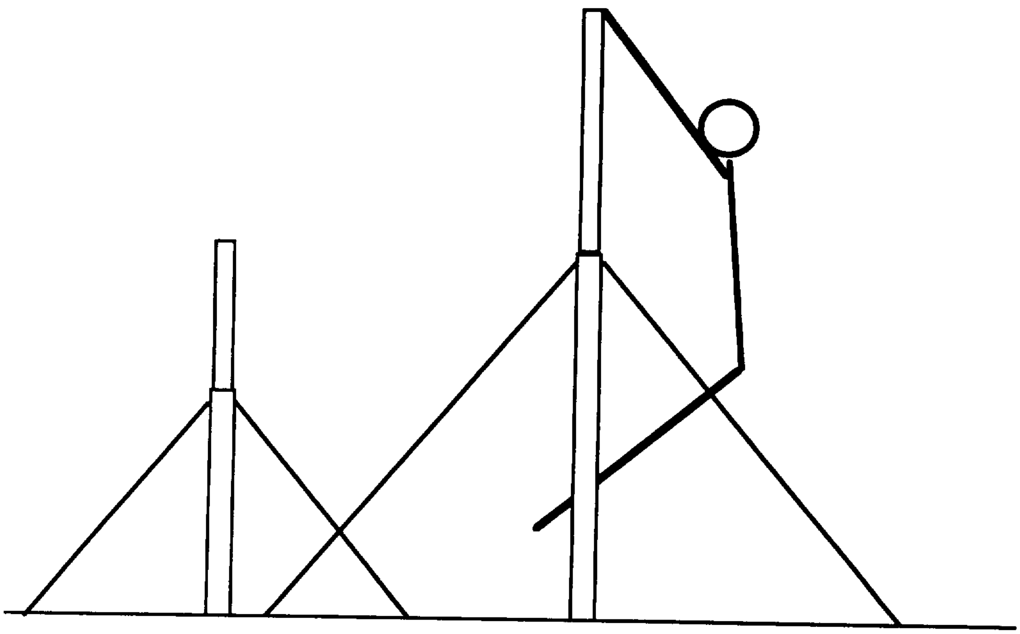
GEORGIA INSTITUTE OF TECHNOLOGY
GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENGINEERING

COMPUTER-AIDED DESIGN
PH.D. QUALIFYING EXAM

FALL 1996

Bras, Fulton, Rosen (Chair)

Ah, the US women's gymnastics team took home the team gold medal this summer! For success in the uneven parallel bars event, the apparatus must be designed to be very rigid, except for the bars themselves - some deflection and spring-back is ideal. For this exam we are asking you to tackle three problems associated with the modeling and analysis of the uneven parallel bars apparatus. A schematic is shown below:



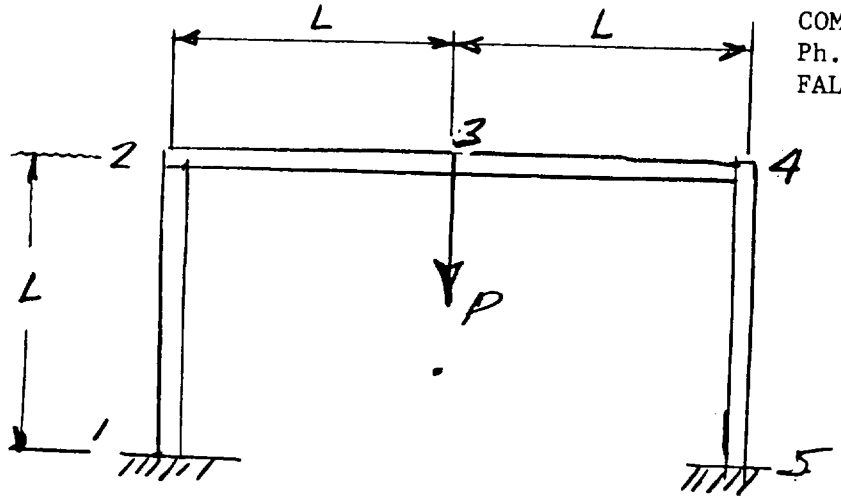
We are interested in learning what you know and your ability to reason. If for some reason you do not follow the question or are confused, kindly adjust the question suitably and proceed with your answer. Please structure your answers as follows:

1) Restate the problem in your own words, identifying any assumptions, judgments, and adjustments that you are making.
2) Tell us your strategy or plan for solving this problem.
3) Solve the problem
4) Tell us about any insight you gained by solving this problem.

Oral Exam Note

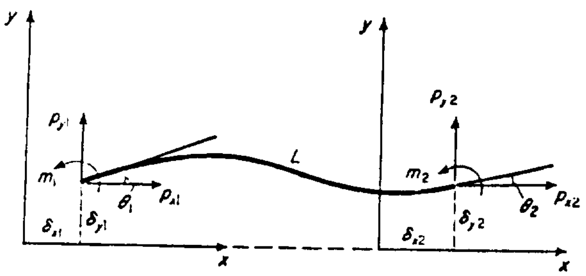
When you come to the oral exam, be prepared to comment briefly on your research activities and where CAE/CAD technology fits into that research.

Problem 1



EI = Constant

A gymnastic bar has the indicated lengths and uniform cross-section properties and Youngs modulus. It is fixed at points 1 and 5. Assuming points 2 and 4 do not displace laterally or vertically, determine the displacement of the bar at point 3 where the gymnast of weight P is hanging. The bending stiffness matrix of a beam is given below for the indicated coordinate system.



Coordinate system for a uniform member in plane bending.

$$\begin{bmatrix} p_{x1} \\ p_{y1} \\ m_1 \end{bmatrix} = \begin{bmatrix} EA/L & 0 & 0 \\ 0 & 12EI/L^3 & 6EI/L^2 \\ 0 & 6EI/L^2 & 4EI/L \end{bmatrix} \begin{bmatrix} \delta_{x1} \\ \delta_{y1} \\ \theta_1 \end{bmatrix} + \begin{bmatrix} -EA/L & 0 & 0 \\ 0 & -12EI/L^3 & 6EI/L^2 \\ 0 & -6EI/L^2 & 2EI/L \end{bmatrix} \begin{bmatrix} \delta_{x2} \\ \delta_{y2} \\ \theta_2 \end{bmatrix}$$

$$\begin{bmatrix} p_{x2} \\ p_{y2} \\ m_2 \end{bmatrix} = \begin{bmatrix} -EA/L & 0 & 0 \\ 0 & -12EI/L^3 & -6EI/L^2 \\ 0 & 6EI/L^2 & 2EI/L \end{bmatrix} \begin{bmatrix} \delta_{x1} \\ \delta_{y1} \\ \theta_1 \end{bmatrix} + \begin{bmatrix} EA/L & 0 & 0 \\ 0 & 12EI/L^3 & -6EI/L^2 \\ 0 & -6EI/L^2 & 4EI/L \end{bmatrix} \begin{bmatrix} \delta_{x2} \\ \delta_{y2} \\ \theta_2 \end{bmatrix}$$

Problem 2

A gymnast performing a routine on the uneven bars slams into the bars with great force. Typically, the bar flexes and an oscillatory movement occurs. Eventually, the movement dampens. For purposes of this investigation, the behavior of a bar can be modeled as a mass on a spring with a damper attached.

$$\frac{d^2x}{dt^2} + \frac{c}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

If the gymnast hits the bar at t=0, such that it is displaced from equilibrium with $x = x_0$ and $dx/dt=0$, then this second-order linear differential problem can be solved using regular means, resulting in the following equation describing the bar's behavior:

$$x(t) = e^{-nt}(x_0 \cos pt + x_0 \frac{n}{p} \sin pt)$$

where $n = c/(2m)$ and $p = \sqrt{(k/m - c^2/(4m^2))}$ and $k/m > c^2/(4m^2)$. Important to note is that the angle term pt is in <u>radians</u>.

Assume that $c = 1.4\text{x}10^7$, $m = 1.2\text{x}10^6$, $k=1.25\text{x}10^9$ and $x_0 = 0.3$.

Questions:

a) Using the false position method, find the time t at which the bar passes for the first time through the equilibrium after the gymnast hits (and excites) the bar. Use t = 0.0 as a lower guess and t = 0.1 as an upper guess. For the sake of time, perform only 3 iterations (no more, no less).

b) Without doing so, if you solved the same problem using the Newton-Rapson method instead of the false position method, would you expect the Newton-Rapson method to have faster or slower convergence for this problem? Why?

c) In comparing the bisection method with Newton-Rapson, list two issues which the bisection method clearly has in favor of Newton Rapson.

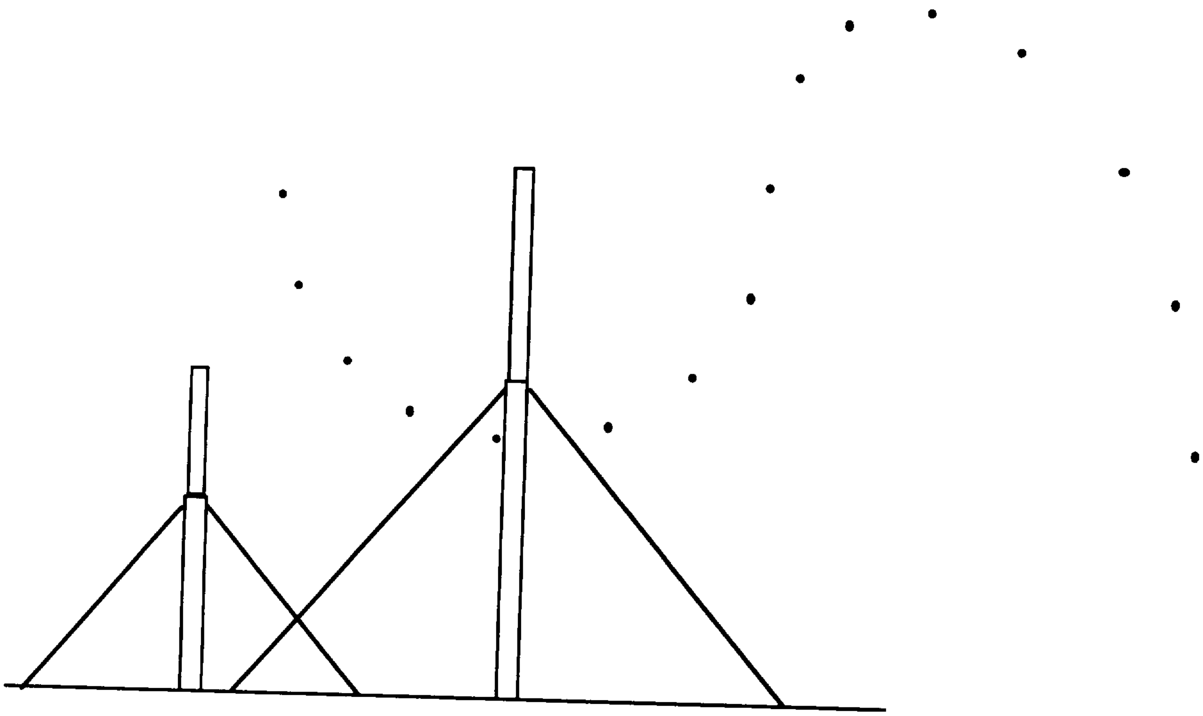
d) Explain the difference between bracketing methods and open techniques.

e) List at least seven different factors, or trade-offs, that should be considered when selecting a numerical method for a particular problem.

3 Geometric Modeling

... and the gymnast is performing her dismount ... Wow, she really stuck that landing! In the figure below, a trace of the gymnast's center of gravity is shown as a series of points in a plane. The points start during a swing around the bar and end with the gymnast standing on the floor after landing. During her flight, she performs two flips and a half-twist.

Please note: the points are intended to be taken at equal time intervals. However, do not try to interpret the shapes too much from my representation. This is just a sketch.



1 Describe the characteristics of the gymnast's path as illustrated by the given points. What order curves would appropriately model these points? Please state your assumptions, particularly about the gymnast's body positions in the air.

2 There are many curve formulations that can be fit to a set of points, including cubic spline, conics (circles, parabolas, ellipses, etc.), parametric curves (Bezier, B-Spline), composite parametric curves, and others. Select three specific curve formulations that could be used to model the given data points. Discuss their advantages and disadvantages in this specific problem. Be specific and comprehensive in your evaluation criteria.

3 Fit a parabola to these three points:
1 (-2, 3)
2 (1, 9)
3 (6, 9)

4 Describe the procedure you would use to fit conics to these data. Be specific regarding (1) which points you are fitting to which curves, and (2) numerical and geometric techniques you would use.