

FEB 18 2003

RESERVE DESK

GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Semester 2002

Design
EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

- Please sign your name on the back of this page—

GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY

DESIGN QUALIFIER

SPRING 2002

WRITTEN EXAMINATION

We are interested in learning what you know and your ability to reason in the formulation and solution of design problems.

If you find any question or part of this exam confusing, please state your assumptions and rephrase the question and proceed.

Please read the entire exam first.

Questions 1 and 2 carry equal points.

Allocate your time carefully so that you cover all three parts that you are being examined on in these two questions, namely, Methods, Realizability and Analysis.

ORAL EXAMINATION

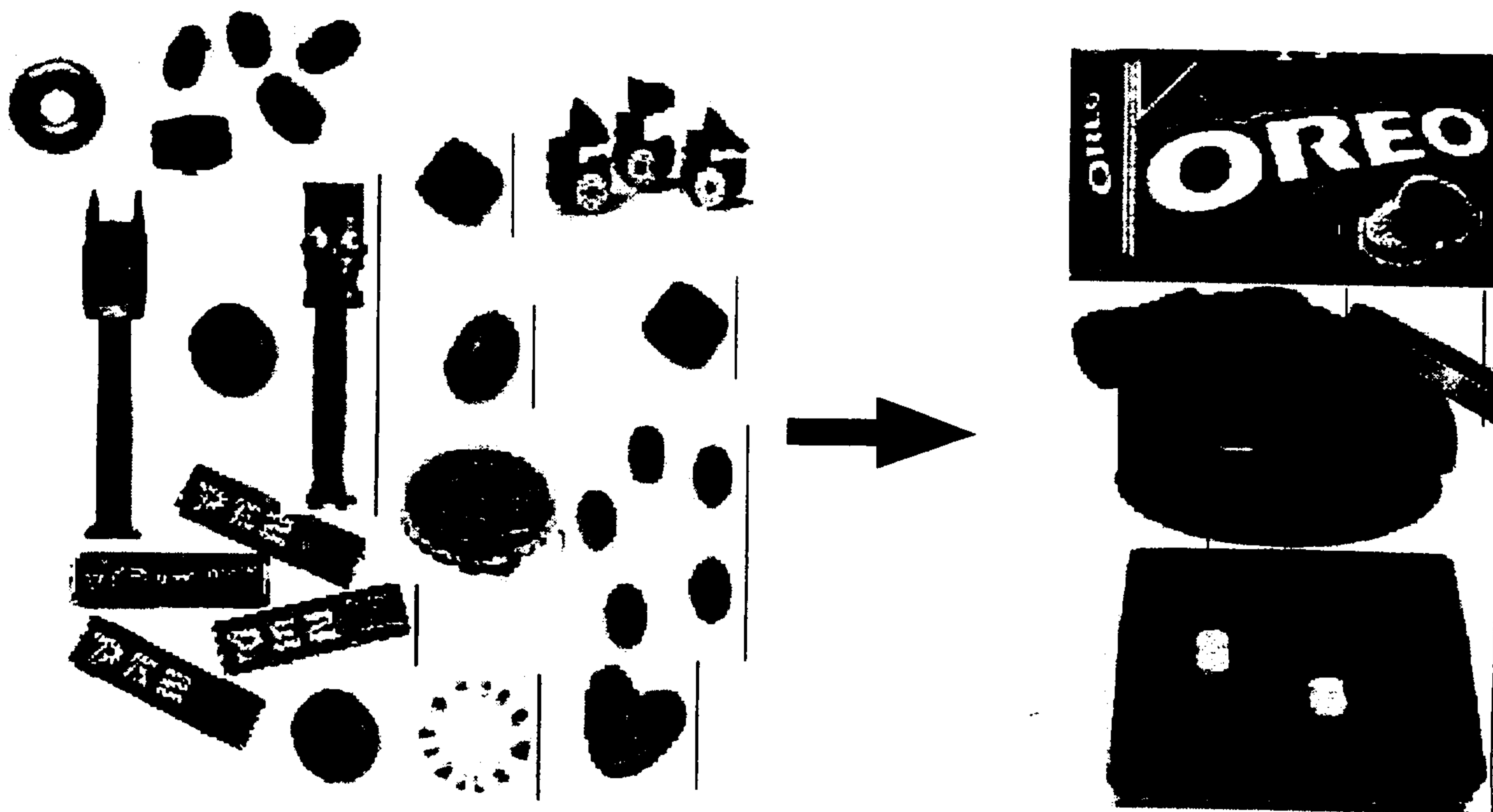
Please arrive half an hour before the scheduled time for the oral exam. During this period we will give you a question to think about. The scope of the oral exam is as follows:

- provide an opportunity for you to state how design fits into your research activities;
- probe your understanding of the question that we posed to you in the preceding half hour;
- reexamine elements of the written exam.

QUESTION 1 - METHOD & REALIZABILITY

Scenario

The mass production of cookies and candy is an important industry that brings joy to millions of people around the world. The treats are produced by a wide variety of processes; however, many of these food items are sorted and packaged in a similar manner. The finished product is transported away from the production area on a conveyor belt. During this process they are examined to make sure they are of acceptable quality. They should not have excess chocolate splattered on them or they should not be cracked, broken, or otherwise visually unappealing. In many cases candy inspectors are used to remove the poorly formed product. The acceptable product is combined into desired lot sizes and then packaged for shipment. The unacceptable product is repaired or recycled. For example, sometimes a candy inspector will simply reshape a sandwich cookie and send it on through the process.



Task

The last five phases of production; 1) transport, 2) inspection, 3) sorting, 4) packaging, and 5) recycling are starting to become more automated. Your task is to design a system that is capable of accomplishing these tasks for a wide range of cookies and candies.

Your boss wants you to start from scratch and document your design process thoroughly - but this is not possible for lack of time. A senior engineer has suggested that you follow the general guidelines given below and turn in a report documenting the five steps.

Deliverables

Method

1. ***Clarify the Task:*** State the overall function of your system. What are the most important drivers/design criteria?
2. ***Conceptual Design:*** State and implement the steps (including a specification list and functional diagrams/decomposition) for transforming the overall function that you have identified into at least three alternative design solutions. Ensure that you have identified the important sub functions for each of the five phases listed above. Sketch and describe the workings of these alternatives.
3. ***Selection:*** Suggest a structured approach to select one of the alternatives for further development.

Realizability

4. ***Embodiment:*** Further develop the alternative that you have selected.
5. ***Costing:*** How would you estimate the cost of your design? You may critically evaluate the design in terms of manufacturability, initial cost, maintenance cost, reliability, manipulation performance, and other criteria that you feel are important to consider in this phase of design.
6. ***Pricing:*** Based on the preceding analysis, how would you estimate the market size for such a system and set the price for selling such a system? Be brief.

Your Exam #:

You MUST write your solutions to QUESTION 2 on this exam sheet.

QUESTION 2 – ANALYSIS

QUESTION 2A

Design a straight-ended helical torsion spring for a static load of 50 N-m at a deflection of 60 ° with a safety factor of 2.

Please state all assumptions.

What are the principal parameters that you, the design engineer, will need to specify for your colleagues, the manufacturing engineer?

Please provide a labeled sketch of the spring.

Determine the design parameters based on the following information:

- 1. Use unpeened, oil-tempered wire.**
- 2. The coil is loaded to close it.**
- 3. Spring index is 10.**
- 4. Spring material is ASTM A229 wire.**
- 5. Length of ends: $L_1 = 40$ mm, $L_2 = 40$ mm**
- 6. Information from Pages 8 through 14.**

Start Question 2B on Next Page

e. As a mechanical designer, which of these conditions would you prefer and why? A load applied gradually or an equal load applied suddenly on a machine part?

f. Can you use stress-concentration factor with ductile materials when they are subject only to static load? Please justify your answer.

Pages 8 through 14 follow.

Important Equations Used in This Chapter

Spring Rate (Section 13.1):

$$k = \frac{F}{y} \quad (13.1)$$

Combining Springs in Parallel (Section 13.1):

$$k_{total} = k_1 + k_2 + k_3 + \dots + k_n \quad (13.2a)$$

Combining Springs in Series (Section 13.1):

$$\frac{1}{k_{total}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \dots + \frac{1}{k_n} \quad (13.2b)$$

Spring Index (Section 13.4):

$$C = \frac{D}{d} \quad (13.5)$$

Deflection of Helical Compression Spring (Section 13.4):

$$y = \frac{8FD^3 N_a}{d^4 G} \quad (13.6)$$

Deflection of Helical Extension Spring (Section 13.7):

$$y = \frac{8(F - F_i)D^3 N_a}{d^4 G} \quad (13.22)$$

Deflection of Round-Wire Helical Torsion Spring (Section 13.8):

$$\theta_{rev} \approx 10.8 \frac{MDN_a}{d^4 E} \quad (13.27c)$$

Spring Rate of Helical Compression Spring (Section 13.4):

$$k = \frac{F}{y} = \frac{d^4 G}{8D^3 N_a} \quad (13.7)$$

Spring Rate of Helical Extension Spring (Section 13.7):

$$k = \frac{F - F_i}{y} = \frac{d^4 G}{8D^3 N_a} \quad (13.20)$$

Spring Rate of Round-Wire Helical Torsion Spring (Section 13.8):

$$k = \frac{M}{\theta_{rev}} \approx \frac{d^4 E}{10.8DN_a} \quad (13.28)$$

Static Stress in Helical Compression or Extension Spring (Section 13.7):

$$\tau_{max} = K_s \frac{8FD}{\pi d^3} \quad \text{where } K_s = \left(1 + \frac{0.5}{C}\right) \quad (13.8b)$$

Dynamic Stress in Helical Compression or Extension Spring (Section 13.7):

$$K_s = \frac{4C + 2}{4C - 3} \quad K_w = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} \quad (13.9a)$$

$$\tau_{max} = K_w \frac{8FD}{\pi d^3} \quad (13.9b)$$

Stress in Helical Torsion Spring at Inside Diameter (Section 13.8):

$$K_{b_i} = \frac{4C^2 - C - 1}{4C(C-1)} \quad (13.31a)$$

$$\sigma_{i_{max}} = K_{b_i} \frac{M_{max} C}{I} = K_{b_i} \frac{M_{max} (d/2)}{\pi d^4 / 64} = K_{b_i} \frac{32 M_{max}}{\pi d^3} \quad (13.32a)$$

Stress in Helical Torsion Spring at Outside Diameter (Section 13.8):

$$K_{b_o} = \frac{4C^2 + C - 1}{4C(C+1)} \quad (13.31b)$$

$$\sigma_{o_{min}} = K_{b_o} \frac{32 M_{min}}{\pi d^3}; \quad \sigma_{o_{max}} = K_{b_o} \frac{32 M_{max}}{\pi d^3} \quad (13.32b)$$

Ultimate Tensile Strength of Steel Wire—See Table 13-4 for Constants (Section 13.4):

$$S_M \cong A d^b \quad (13.3)$$

Ultimate Shear Strength of Wire (Section 13.4):

$$S_{MS} \cong 0.67 S_M \quad (13.4)$$

Torsional Endurance Limits for Spring-Steel Wire for Stress Ratio $R = 0$ (Section 13.4):

$$\begin{aligned} S_{sw} &\cong 45.0 \text{ kpsi (310 MPa)} \quad \text{for unpeened springs} \\ S_{sw} &\cong 67.5 \text{ kpsi (465 MPa)} \quad \text{for peened springs} \end{aligned} \quad (13.12)$$

Torsional Endurance Limits for Spring-Steel Wire for Stress Ratio $R = -1$ (Section 13.4):

$$S_{es} = 0.5 \frac{S_{sw} S_{MS}}{S_{MS} - 0.5 S_{sw}} \quad (13.17b)$$

Bending Endurance Limits for Spring-Steel Wire for Stress Ratio $R = 0$ (Section 13.4):

$$S_{sw_b} = \frac{S_{sw}}{0.577} \quad (13.33a)$$

Bending Endurance Limits for Spring-Steel Wire for Stress Ratio $R = -1$ (Section 13.4):

$$S_e = 0.5 \frac{S_{sw_b} S_M}{S_M - 0.5 S_{sw_b}} \quad (13.34c)$$

Static Safety Factor for Helical Compression or Extension Spring (Section 13.5):

$$N_s = \frac{S_M}{\tau} \quad (13.14)$$

Dynamic Safety Factor for Helical Compression or Extension Spring (Section 13.4):

$$N_{fs} = \frac{S_{es} (S_M - \tau_l)}{S_{es} (\tau_m - \tau_l) + S_M \tau_m} \quad (13.17a)$$

Dynamic Safety Factor for Helical Torsion Spring (Section 13.8):

$$N_{fb} = \frac{S_e (S_M - \sigma_{o_{min}})}{S_e (\sigma_{o_{max}} - \sigma_{o_{min}}) + S_M \sigma_{o_{min}}} \quad (13.34b)$$

The stresses σ_a and σ_m can replace S_a and S_m in Eqs. (7-34) to (7-36) if each strength is divided by a factor of safety n . When this is done, the Soderberg equation becomes

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}$$

The modified Goodman relation is

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{wm}} = \frac{1}{n}$$

and the Gerber equation is

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{wm}}\right)^2 = 1$$

The critical deflection is given by the equation

$$y_{cr} = L_0 C_1 \left[1 - \left(1 - \frac{C_2}{\lambda_{eff}^2} \right)^{1/2} \right]$$

where y_{cr} is the deflection corresponding to the onset of instability

TABLE 10-2
Formulas for Compression-Spring Dimensions. (N_a = Number of Active Coils)

TERM	TYPE OF SPRING ENDS			
	PLAIN	PLAIN AND GROUND	SQUARED OR CLOSED	SQUARED AND GROUND
End coils, N_e	0	1	2	2
Total coils, $N_t \cong N$	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, $L_0 \cong L_f$	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

Source: Associated Spring-Barnes Group, Design Handbook, Bristol, Conn., 1981, p. 32.

TABLE 10-3
End-Condition Constants α for Helical Compression Springs*

END CONDITION	CONSTANT α
Spring supported between flat parallel surfaces (fixed ends)	0.5
One end supported by flat surface perpendicular to spring axis (fixed); other end pivoted (hinged)	0.707
Both ends pivoted (hinged)	1
One end clamped; other end free	2

*Ends supported by flat surfaces must be squared and ground.

quantity λ_{eff} in Eq. (10-11) is the *effective slenderness ratio* and is given by the equation

$$\lambda_{eff} = \frac{\alpha L_0}{D} \quad (10-12)$$

C_1 and C_2 are the elastic constants and are defined by the equations

$$C_1 = \frac{E}{2(E - G)} \quad (10-13)$$

$$C_2 = \frac{2\pi^2(E - G)}{2G + E} \quad (10-14)$$

Equation (10-12) contains the *end-condition constant* α . This depends upon how the ends of the spring are supported. Table 10-3 gives values of α for usual end conditions. Note how closely these resemble the end conditions for columns.

Absolute stability occurs when, in Eq. (10-11), the term C_2/λ_{eff}^2 is less than unity. This means that the condition for absolute stability is that

$$L_0 < \frac{\pi D}{\alpha} \left[\frac{2(E - G)}{2G + E} \right]^{1/2} \quad (10-15)$$

For steels, this turns out to be

$$L_0 < 2.63 \frac{D}{\alpha} \quad (10-16)$$

Table 13-5 Typical Properties of Spring Temper Alloy Strip

Source: Reference 1

Material	Sut MPa (ksi)	Rockwell Hardness	Elongation %	Bend Factor	E GPa (Mpsi)	Poisson Ratio
Spring steel	1 700 (246)	C50	2	5	207 (30)	0.30
Stainless 301	1 300 (189)	C40	8	3	193 (28)	0.31
Stainless 302	1 300 (189)	C40	5	4	193 (28)	0.31
Monel 400	690 (100)	B95	2	5	179 (26)	0.32
Monel K500	1 200 (174)	C34	40	5	17.9 (26)	0.29
Inconel 600	1 040 (151)	C30	2	2	214 (31)	0.29
Inconel X-750	1 050 (152)	C35	20	3	214 (31)	0.29
Beryllium copper	1 300 (189)	C40	2	5	128 (18.5)	0.33
Ni-Span-C	1 400 (203)	C42	6	2	186 (27)	-
Brass CA 260	620 (90)	B90	3	3	11 (16)	0.33
Phosphor bronze	690 (100)	B90	3	2.5	103 (15)	0.20
17-7PH RH950	1 450 (210)	C44	6	flat	203 (29.5)	0.34
17-7PH Cond. C	1 650 (239)	C46	1	2.5	203 (29.5)	0.34

$$F_a = \frac{F_{max} - F_{min}}{2}$$

$$F_m = \frac{F_{max} + F_{min}}{2}$$

A force ratio R_F can also be defined as:

$$R_F = \frac{F_{min}}{F_{max}}$$

Table 13-4 Coefficients and Exponents for Equation 13.3

Source: Reference 1

ASTM #	Material	Range		Exponent b	Coefficient A		Correlation Factor
		mm	in		MPa	psi	
A227	Cold drawn	0.5-16	0.020-0.625	-0.182 2	1 753.3	141 040	0.998
A228	Music wire	0.3-6	0.010-0.250	-0.1625	2 153.5	184 649	0.9987
A229	Oil tempered	0.5-16	0.020-0.625	-0.183 3	1 831.2	146 780	0.999
A232	Chrome-v.	0.5-12	0.020-0.500	-0.145 3	1 909.9	173 128	0.998
A401	Chrome-s.	0.8-11	0.031-0.437	-0.093 4	2 099.2	220 779	0.991

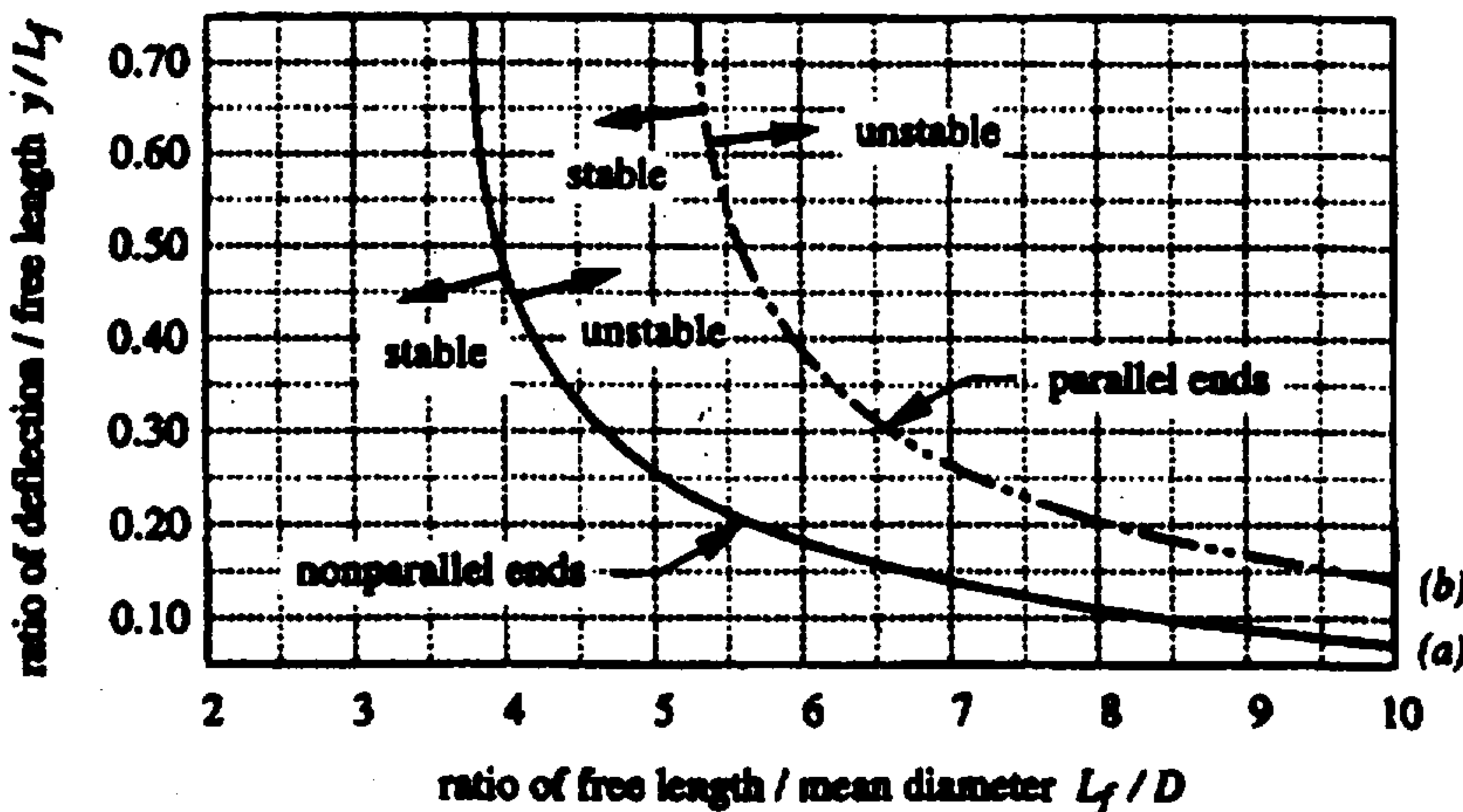


FIGURE 13-14

Critical Buckling Condition Curves Adapted from Reference 1

Substituting equations 13.7 (p. 810) and 13.11a into 13.11b gives

$$f_n = \frac{2}{\pi N_a} \frac{d}{D^2} \sqrt{\frac{Gg}{32\gamma}} \text{ Hz} \quad (13.11c)$$

for the natural frequency of a fixed-fixed helical coil spring. If one end of the spring is fixed and the other free, it acts like a fixed-free spring of twice its length. Its natural frequency can be found by using a number for N_a in equation 13.11c that is twice the actual number of active coils present in the fixed-free spring.

Table 13-6 Maximum Torsional Yield Strength S_{ys} for Helical Compression Springs in Static Applications

Bending or Buckling Stresses Not Included. Source: Adapted from Ref. 1

Material	Maximum Percent of Ultimate Tensile Strength	
	Before Set Removed (Use Eq. 13.9b)	After Set Removed (Use Eq. 13.9c)
Cold-drawn carbon steel (e.g., A227, A228)	50	60-70%
Hardened and tempered carbon and low-alloy steel (e.g., A229, A230, A232, A401)	50	65-75
Austenitic stainless steel (e.g., A313)	35	55-65
Nonferrous alloys (e.g., B134, B159, B197)	35	55-65

$$S_{ys} = 45\% S_{ut}$$

Table 13-2

Preferred Wire Diameters

U.S. (In)	SI (mm)
0.004	0.10
0.005	0.12
0.006	0.16
0.008	0.20
0.010	0.25
0.012	0.30
0.014	0.35
0.016	0.40
0.018	0.45
0.020	0.50
0.022	0.55
0.024	0.60
0.026	0.65
0.028	0.70
0.030	0.80
0.035	0.90
0.038	1.00
0.042	1.10
0.045	
0.048	1.20
0.051	
0.055	1.40
0.059	
0.063	1.60
0.067	
0.072	1.80
0.076	
0.081	2.00
0.085	2.20
0.092	
0.098	2.50
0.105	
0.112	2.80
0.125	3.00
0.135	3.50
0.148	
0.162	4.00
0.177	4.50
0.192	5.00
0.207	5.50
0.225	6.00
0.250	6.50
0.281	7.00
0.312	8.00
0.343	9.00
0.362	
0.375	
0.406	10.0
0.437	11.0
0.469	12.0
0.500	13.0
0.531	14.0
0.562	15.0
0.625	16.0

Active Coils in Extension Springs

All coils in the body are considered active coils, but one coil is typically added to the number of active coils to obtain the body length L_b .

$$N_t = N_a + 1 \tag{13.18}$$

$$L_b = dN_t \tag{13.19}$$

$$\tau_i \approx -4.231C^3 + 181.5C^2 - 3387C + 28640 \tag{13.21a}$$

$$\tau_i \approx -2.987C^3 + 139.7C^2 - 3427C + 38404 \tag{13.21b}$$

where τ_i is in psi. The average of the two values computed from these functions can be taken as a good starting value for initial coil stress.

The bending stress at point A is found from

$$\sigma_A = K_b \frac{16DF}{\pi d^3} + \frac{4F}{\pi d^2}$$

where

$$K_b = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)}$$

and

$$C_1 = \frac{2R_1}{d}$$

The torsional stress at point B is found from

$$\tau_B = K_{W_2} \frac{8DF}{\pi d^3} \tag{13.24a}$$

where

$$K_{W_2} = \frac{4C_2 - 1}{4C_2 - 4} \tag{13.24b}$$

and

$$C_2 = \frac{2R_2}{d} \tag{13.24c}$$

R_2 is the side-bend radius, as shown in Figure 13-23. C_2 should be greater than 4.^[1]

Surging in Extension Springs

The natural frequency of a helical extension spring with both ends fixed against axial deflection is the same as that for a helical spring in compression (see equation 13.11):

$$f_n = \frac{2}{\pi N_a} \frac{d}{D^2} \sqrt{\frac{Gg}{32\gamma}} \text{ Hz} \tag{13.25}$$

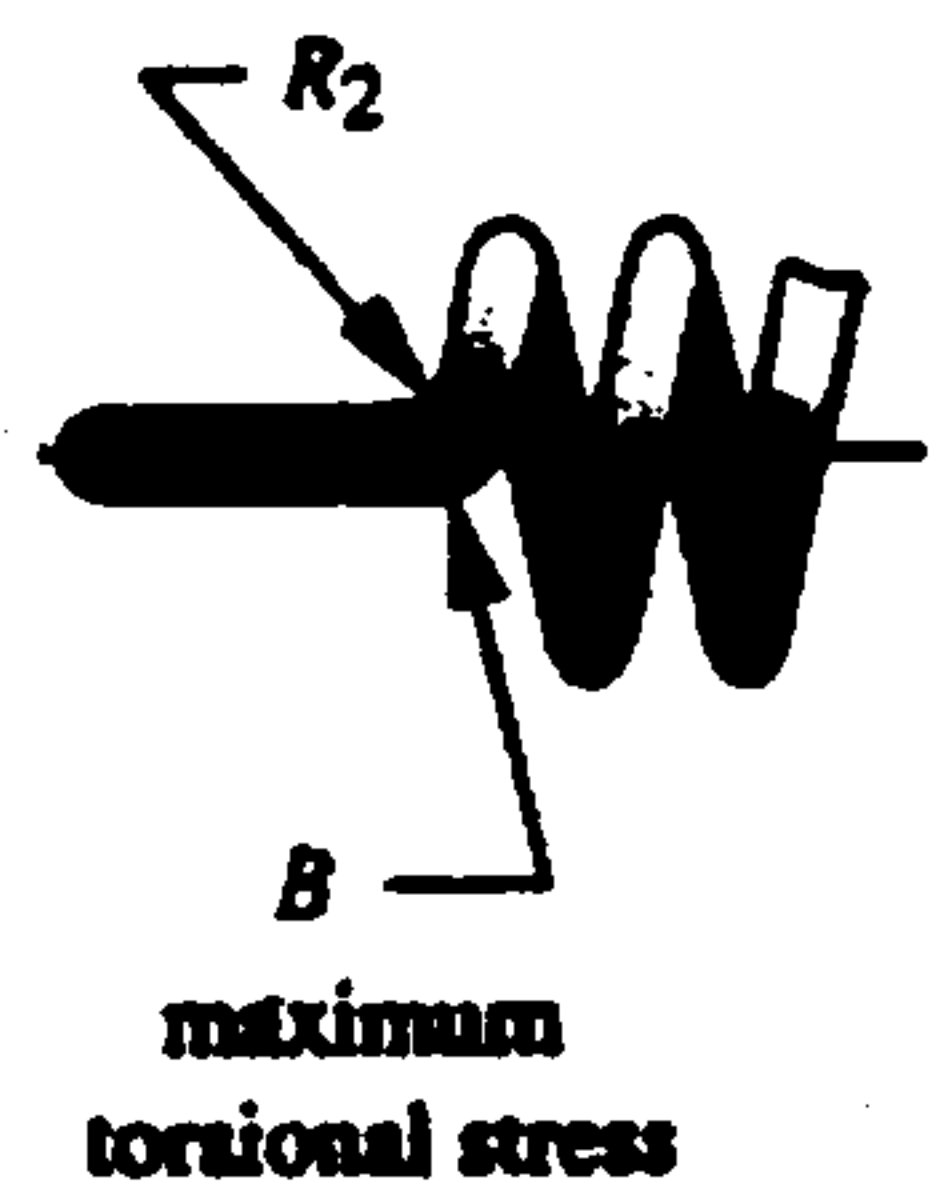
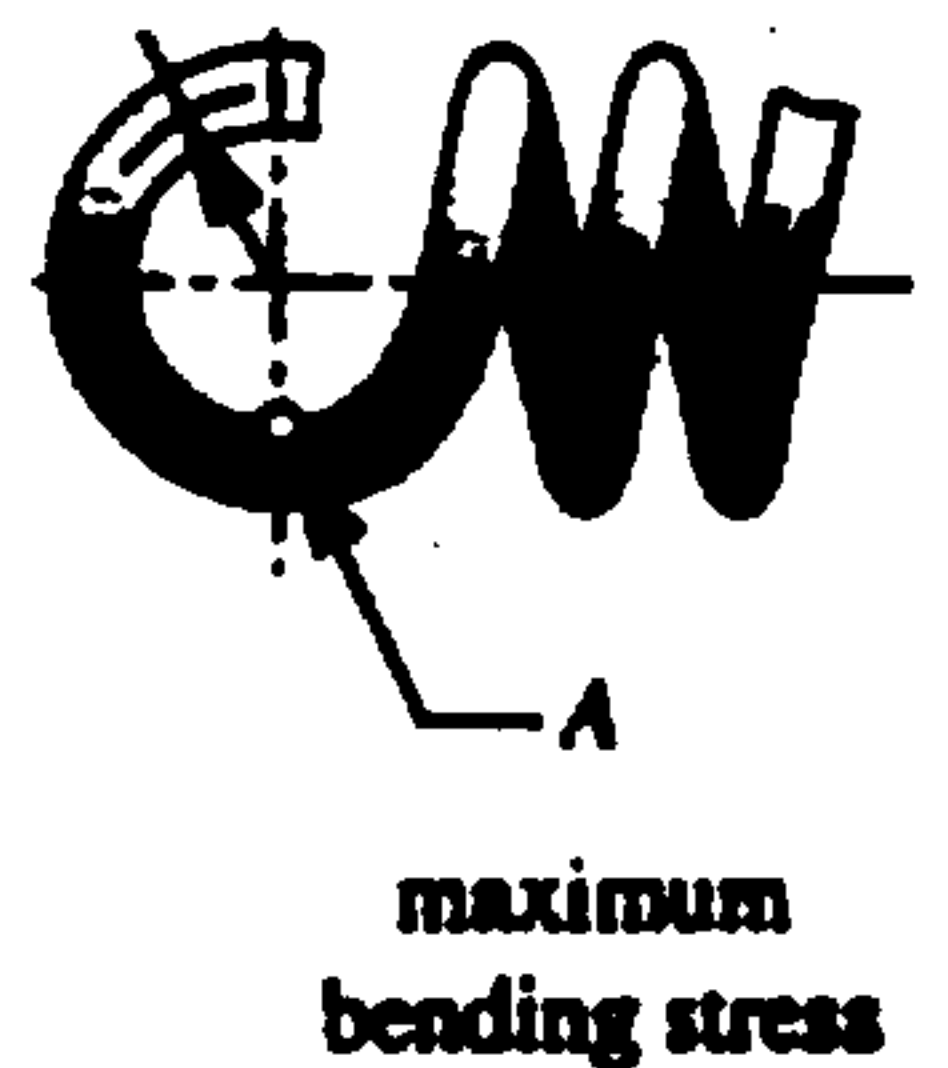


Table 13-10 Maximum Torsional and Bending Yield Strengths S_{ys} and S_y for Helical Extension Springs in Static Applications

No Set Removal and Low-Temperature Heat Treatment Applied. Source: Ref. 1

Material	Maximum Percent of Ultimate Tensile Strength		
	S_{ys} in Torsion		S_y in Bending
	Body	End	End
Cold-drawn carbon steel (e.g., A227, A228)	45%	40%	75%
Hardened and tempered carbon and low-alloy steel (e.g., A229, A230, A232, A401)	50	40	75
Austenitic stainless steel and nonferrous alloys (e.g., A313, B134, B199, B197)	35	30	55

Number of Coils in Torsion Springs

The active coils are equal to the number of body turns N_b plus some contribution from the ends, which also bend. For straight ends, the contribution can be expressed as an equivalent number of coils N_e :

$$N_e = \frac{L_1 + L_2}{3\pi D} \quad (13.26a)$$

where L_1 and L_2 are the respective lengths of the tangent-ends of the coil. The number of active coils is then

$$N_a = N_b + N_e \quad (13.26b)$$

where N_b is the number of coils in the spring body.

Deflection of Torsion Springs

The angular deflection of the coil-end is normally expressed in radians but is often converted to revolutions. We will use revolutions. Since it is essentially a beam in bending, the (angular) deflection can be expressed as

$$\theta_{rev} = \frac{1}{2\pi} \theta_{rad} = \frac{1}{2\pi} \frac{ML_w}{EI} \quad (13.27a)$$

Coil Closure

When a torsion spring is loaded to close the coils (as it should be), the coil is "wound up." The minimum inside coil diameter at full deflection is

$$D_{i_{min}} = \frac{DN_b}{N_b + \theta_{rev}} - d \quad (13.29)$$

where D is the unloaded mean coil diameter. Any pin that the coil works over should be limited to about 90% of this minimum inside diameter.

The maximum coil-body length at full "windup" is:

$$L_{max} = d(N_b + 1 + \theta) \quad (13.30)$$

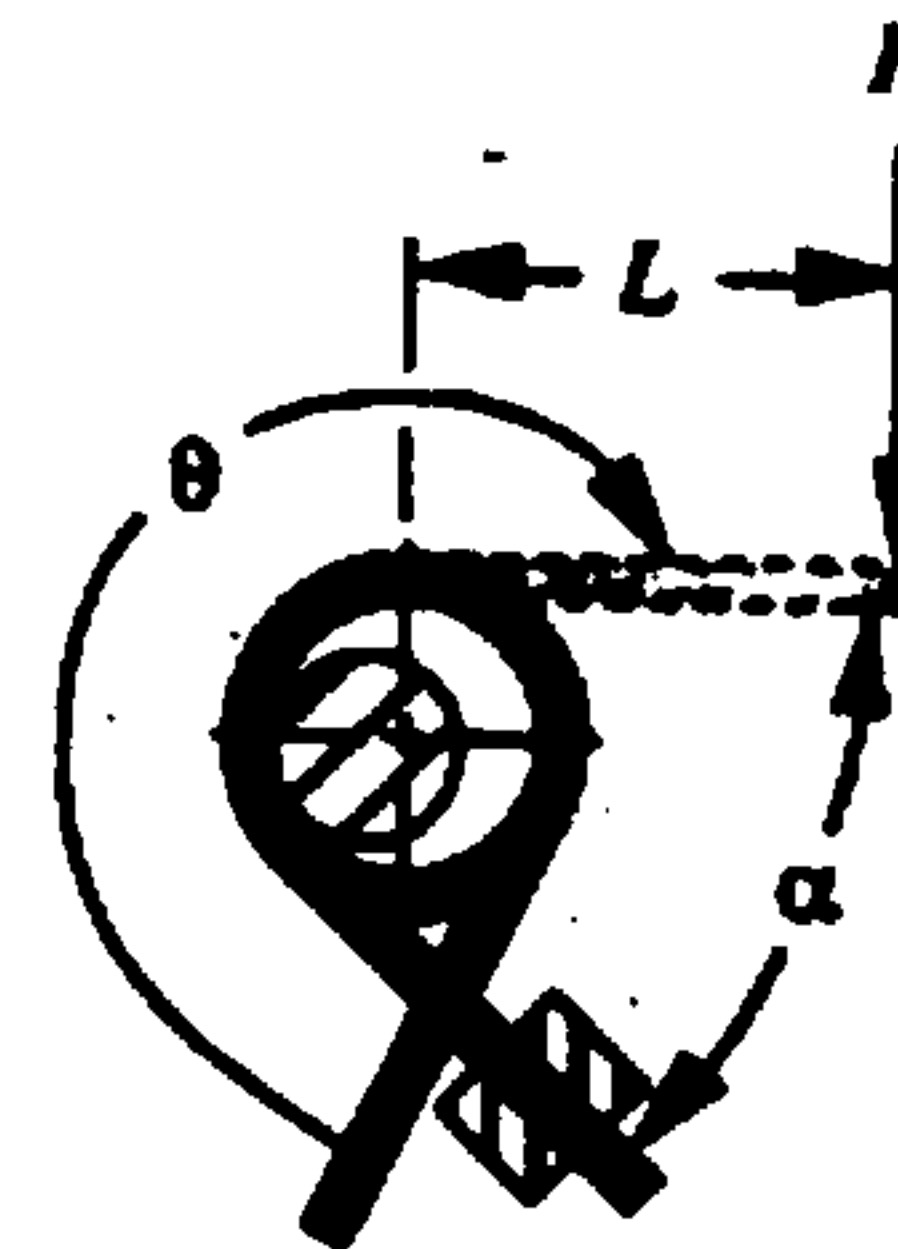
The torsional-endurance limit data for helical compression springs shown in equation 13.12 (p. 816) can be adapted for bending by using the von Mises relationship between torsion and tension loading.

$$S_{sw_b} = \frac{S_{sw_t}}{0.577} \quad (13.33a)$$

which gives

$$S_{sw_b} \cong \frac{45.0}{0.577} = 78 \text{ kpsi (537 MPa) for unpeened springs} \quad (13.33b)$$

$$S_{sw_b} \cong \frac{67.5}{0.577} = 117 \text{ kpsi (806 MPa) for peened springs}$$



free position

specify:

α —angle between ends

F —load on ends at α

L —moment arm

θ —angular deflection

from free position

Table 13-13 Maximum Recommended Bending Yield Strength S_y for Helical Torsion Springs in Static Applications
Source: Adapted from Reference 1

Material	Maximum Percent of Ultimate Tensile Strength	
	Stress Relieved	Favorable Residual Stress
Cold-drawn carbon steel (e.g., A227, A228)	80%	100%
Hardened and tempered carbon and low-alloy steel (e.g., A229, A230, A232, A401)	85	100
Austenitic stainless steel and nonferrous alloys (e.g., A313, B134, B159, B197)	60	80

Table C-1 Physical Properties of Some Engineering Materials

Data from Various Sources.* These Properties are Essentially Similar for All Alloys of the Particular Material

Material	Modulus of Elasticity <i>E</i>		Modulus of Rigidity <i>G</i>		Poisson's Ratio ν	Weight Density γ	Mass Density ρ	Specific Gravity
	Mpsi	GPa	Mpsi	GPa		lb/in ³	Mg/m ³	
Aluminum Alloys	10.4	71.7	3.9	26.8	0.34	0.10	2.8	2.8
Beryllium Copper	18.5	127.6	7.2	49.4	0.29	0.30	8.3	8.3
Brass, Bronze	16.0	110.3	6.0	41.5	0.33	0.31	8.6	8.6
Copper	17.5	120.7	6.5	44.7	0.35	0.32	8.9	8.9
Iron, Cast, Gray	15.0	103.4	5.9	40.4	0.28	0.26	7.2	7.2
Iron, Cast, Ductile	24.5	168.9	9.4	65.0	0.30	0.25	6.9	6.9
Iron, Cast, Malleable	25.0	172.4	9.6	66.3	0.30	0.26	7.3	7.3
Magnesium Alloys	6.5	44.8	2.4	16.8	0.33	0.07	1.8	1.8
Nickel Alloys	30.0	206.8	11.5	79.6	0.30	0.30	8.3	8.3
Steel, Carbon	30.0	206.8	11.7	80.8	0.28	0.28	7.8	7.8
Steel, Alloys	30.0	206.8	11.7	80.8	0.28	0.28	7.8	7.8
Steel, Stainless	27.5	189.6	10.7	74.1	0.28	0.28	7.8	7.8
Titanium Alloys	16.5	113.8	6.2	42.4	0.34	0.16	4.4	4.4
Zinc Alloys	12.0	82.7	4.5	31.1	0.33	0.24	6.6	6.6

* Properties of Some Metals and Alloys, International Nickel Co., Inc., N.Y.; Metals Handbook, American Society for Metals, Materials Park, Ohio.