GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Semester 2002

Design
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY

DESIGN QUALIFIER

FALL 2002

WRITTEN EXAMINATION

We are interested in learning what you know and your ability to reason in the formulation and solution of design problems.

If you find any question or part of this exam confusing, please state your assumptions and rephrase the question and proceed.

Please read the entire exam first.

Questions 1 and 2 carry equal points.

Allocate your time carefully so that you cover all three parts that you are being examined on in these two questions, namely, Methods, Realizability and Analysis.

ORAL EXAMINATION

Please arrive half an hour before the scheduled time for the oral exam. During this period we will give you a question to think about. The scope of the oral exam is as follows:

* provide an opportunity for you to state how design fits into your research activities;
* probe your understanding of the question that we posed to you in the preceding half hour.

QUESTION 1 – METHOD & REALIZABILITY

Scenario

Automation in the pharmaceutical industry is becoming more and more important as new and more various drugs are being developed and tested. Automation is critical in this effort. One task that lends itself quite nicely to automation is a multi-volumetric fluid testing system. This is a fancy name for an automated unit that takes anywhere from 10 to several hundred test tubes of chemical and biological agents and adds a variety of chemicals to the test tubes in various amounts. For example, one might wish to add 1 ml – 100 ml of a saline solution to test tubes 1 – 100 in increments of 1 ml, and 1 – 10 ml of a 0.01% hydrochloric acid solution to test tubes 101 – 200 in increments of 0.1 ml. So this system needs to be able to precisely place (within 1 mm) a pipette over a test tube and deliver a liquid into the tube. Figure 1 is a schematic of a Cartesian based system capable of holding 30 test tubes. This figure shows two drive systems. However, there must be some sort of third axis as the system must be able to place and remove test tubes from the holder. Please note that this is one of many configurations; so do not limit yourself to the example shown in Figure 1.

Also, there are a variety of possible uses and users for this system ranging from small university research groups to large pharmaceutical companies, who will test anywhere from 10 test tubes at a time to several thousand at a time. Due to const constraints, you are not permitted to design several systems of different capacities. Rather you need to design a system that is expandable from 10 – 5000 test tubes. It is anticipated that by making the system expandable, you will save costs from several perspectives. First your company will have a small inexpensive system that can handle a small number of test tubes. Second, the expanded system, since it uses the components of the smaller system will be less expensive due to economies of scale. Finally, with a simple expansion kit a customer can expand the capability of their system without having to purchase an entirely new system. This is a great marketing point.

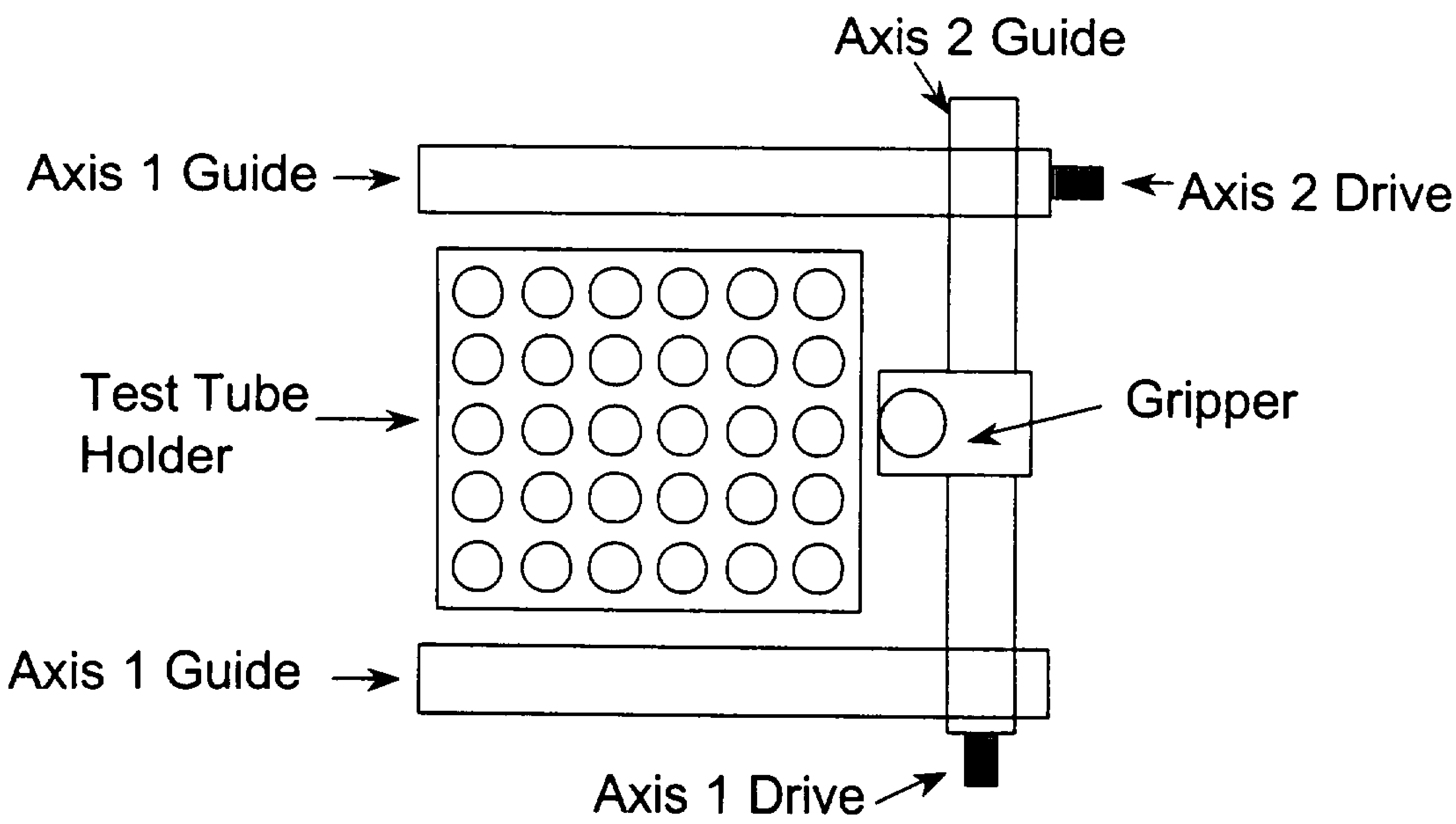


Figure 1: A Cartesian Type Gantry System with a Capacity of 30 Test Tubes.

Task

Your task is to design a system that is capable of positioning the pipette and delivering the fluid at a rate of 1 test tube per 5 seconds. It must be able to test a minimum of 10 samples, but should be expandable to 5000 samples.

Your boss wants you to start from scratch and document your design process thoroughly – but this is not possible for lack of time. A senior engineer has suggested that you follow the general guidelines given below and turn in a report documenting the five steps.

Deliverables

Method

1. *Clarify the Task:* State the overall function of your system. What are the most important drivers/design criteria?

2. *Conceptual Design:* State and **implement** the steps (including a requirements / specification list and functional diagrams/decomposition) for transforming the overall function that you have identified into at least three alternative design solutions. Ensure that you have identified the important sub functions for the operation of the device. Sketch and describe the workings of these alternatives.

3. *Selection:* Suggest a structured approach to select one of the alternatives for further development.

Realizability

4. *Embodiment:* Further develop the alternative that you have selected.

5. *Costing:* How would you estimate the cost of your design? You may critically evaluate the design in terms of manufacturability, initial cost, maintenance cost, reliability, manipulation performance, and other criteria that you feel are important to consider in this phase of design.

6. *Pricing:* Based on the preceding analysis, how would you estimate the market size for such a system and set the price for selling such a system? Be brief.

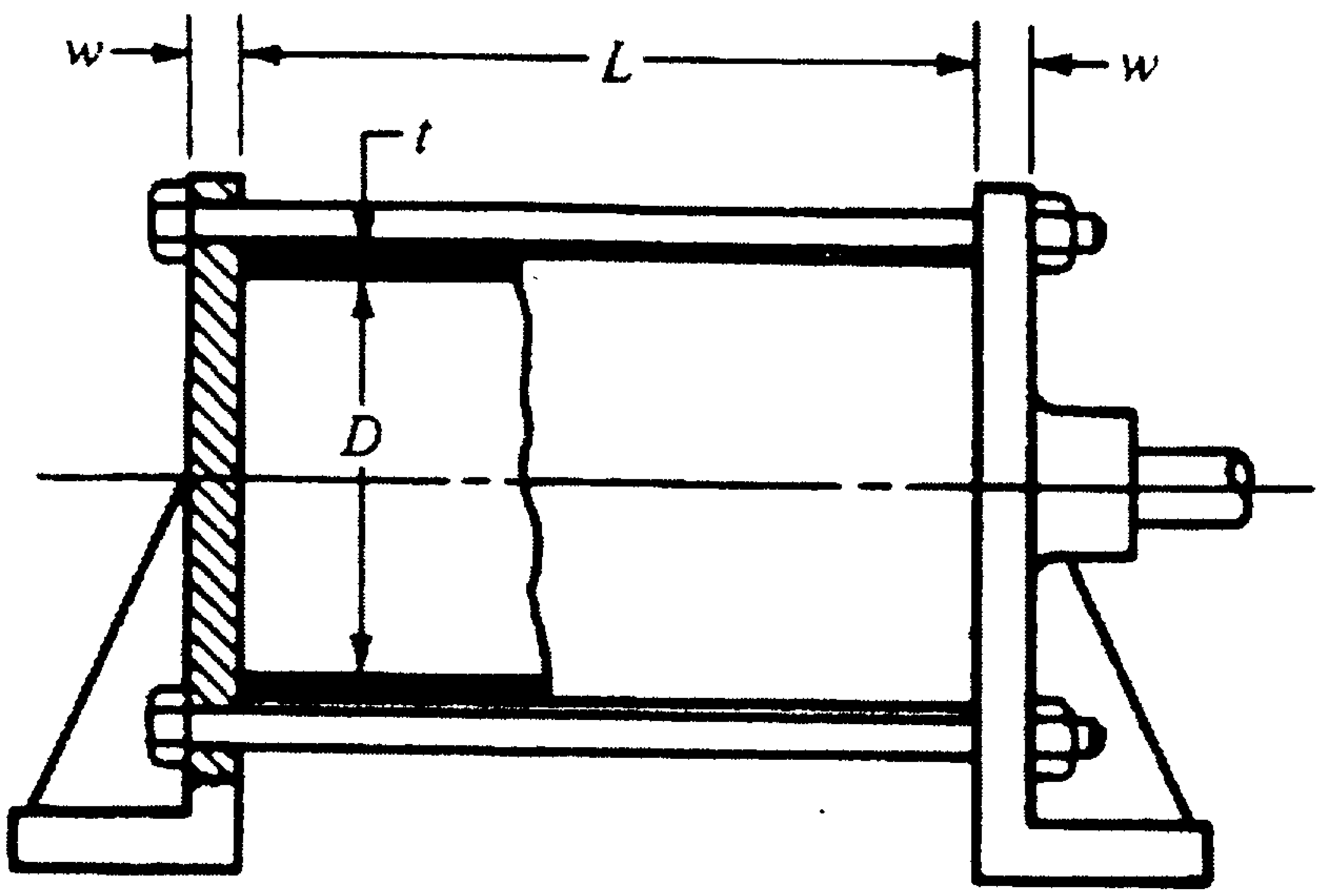
Your Exam #: ....
You MUST write your solutions to QUESTION 2 on this exam sheet.

QUESTION 2 – ANALYSIS

QUESTION 2A

The figure shows a fluid-pressure linear actuator (hydraulic cylinder) in which

> D = 4 in, t = 3/8 in, L = 12 in, and w = 3/4 in. Both brackets as well as the cylinder are of steel. The actuator has been designed for a working pressure of 2000 psi. Six 3/8 -in SAE grade 5 coarse-thread bolts are used, tightened to 75 percent of proof load.



Please state all assumptions.

(a) Find the stiffnesses of the bolts and members, assuming that the entire cylinder is compressed uniformly and that the end brackets are perfectly rigid.
(b) Using the Goodman criterion, find the factor of safety guarding against a fatigue failure.
(c) What pressure would be required to cause total joint separation?

Please make use of the information given on Pages 9 through 13.

Start Question 2B on Next Page

2B. Please answer at least EIGHT of the TEN questions

a. Explain the meaning of L_{10} life with respect to bearings.

b. List four primary reasons why bearings fail.

c. Is buckling an issue with helical extension springs? Please explain.

d. Why are the ends of torsion spring coils extended tangentially?

e. Discuss the major difference between a confined and unconfined gasket.

f. The torque that twists the screw is dependent on what?

g. Discuss two ways to ensure that a bolted joint will not fail when loaded by an external force F.

h. What two modes of failure affect gear teeth? What causes each type of failure?

i. Name two types of gears that can be used for gear trains with nonparallel shafts.

j. In a structure that is experiencing fatigue load, what are S_e and S_e' called? What is the difference between them?

Pages 9 through 13 follow.

Table 14-1 Principal Dimensions of Unified National Standard Screw Threads

Data Calculated from Equations 14-1—See Reference 3 for More Information

		Coarse Threads—UNC			Fine Threads—UNF		
Size	Major Diameter d (in)	Threads per inch	Minor Diameter d_r (in)	Tensile Stress Area A_t (in²)	Threads per inch	Minor Diameter d_r (in)	Tensile Stress Area A_t (in²)
0	0.0600	–	–	–	80	0.0438	0.0018
1	0.0730	64	0.0527	0.0026	72	0.0550	0.0028
2	0.0860	56	0.0628	0.0037	64	0.0657	0.0039
3	0.0990	48	0.0719	0.0049	56	0.0758	0.0052
4	0.1120	40	0.0795	0.0060	48	0.0849	0.0066
5	0.1250	40	0.0925	0.0080	44	0.0955	0.0083
6	0.1380	32	0.0974	0.0091	40	0.1055	0.0101
8	0.1640	32	0.1234	0.0140	36	0.1279	0.0147
10	0.1900	24	0.1359	0.0175	32	0.1494	0.0200
12	0.2160	24	0.1619	0.0242	28	0.1696	0.0258
1/4	0.2500	20	0.1850	0.0318	28	0.2036	0.0364
5/16	0.3125	18	0.2403	0.0524	24	0.2584	0.0581
3/8	0.3750	16	0.2938	0.0775	24	0.3209	0.0878
7/16	0.4375	14	0.3447	0.1063	20	0.3725	0.1187
1/2	0.5000	13	0.4001	0.1419	20	0.4350	0.1600
9/16	0.5625	12	0.4542	0.1819	18	0.4903	0.2030
5/8	0.6250	11	0.5069	0.2260	18	0.5528	0.2560
3/4	0.7500	10	0.6201	0.3345	16	0.6688	0.3730
7/8	0.8750	9	0.7307	0.4617	14	0.7822	0.5095
1	1.0000	8	0.8376	0.6057	12	0.8917	0.6630
1 1/8	1.1250	7	0.9394	0.7633	12	1.0167	0.8557
1 1/4	1.2500	7	1.0644	0.9691	12	1.1417	1.0729
1 3/8	1.3750	6	1.1585	1.1549	12	1.2667	1.3147
1 1/2	1.5000	6	1.2835	1.4053	12	1.3917	1.5810
1 3/4	1.7500	5	1.4902	1.8995			
2	2.0000	4.5	1.7113	2.4982			
2 1/4	2.2500	4.5	1.9613	3.2477			
2 1/2	2.5000	4	2.1752	3.9988			
2 3/4	2.7500	4	2.4252	4.9340			
3	3.0000	4	2.6752	5.9674			
3 1/4	3.2500	4	2.9252	7.0989			
3 1/2	3.5000	4	3.1752	8.3286			
3 3/4	3.7500	4	3.4252	9.6565			
4	4.0000	4	3.6752	11.0826			

Table 14-2 Principal Dimensions of ISO Metric Standard Screw Threads

Data Calculated from Equations 14-1—See Reference 4 for More Information

	Coarse Threads			Fine Threads		
Major Diameter d (mm)	Pitch p mm	Minor Diameter d_r (mm)	Tensile Stress Area A_t (mm²)	Pitch p mm	Minor Diameter d_r (mm)	Tensile Stress Area A_t (mm²)
3.0	0.50	2.39	5.03			
3.5	0.60	2.76	6.78			
4.0	0.70	3.14	8.78			
5.0	0.80	4.02	14.18			
6.0	1.00	4.77	20.12			
7.0	1.00	5.77	28.86			
8.0	1.25	6.47	36.61	1.00	6.77	39.17
10.0	1.50	8.16	57.99	1.25	8.47	61.20
12.0	1.75	9.85	84.27	1.25	10.47	92.07
14.0	2.00	11.55	115.44	1.50	12.16	124.55
16.0	2.00	13.55	156.67	1.50	14.16	167.25
18.0	2.50	14.93	192.47	1.50	16.16	216.23
20.0	2.50	16.93	244.73	1.50	18.16	271.50
22.0	2.50	18.93	303.40	1.50	20.16	333.06
24.0	3.00	20.32	352.50	2.00	21.55	384.42
27.0	3.00	23.32	459.41	2.00	24.55	495.24
30.0	3.50	25.71	560.59	2.00	27.55	621.20
33.0	3.50	28.71	693.55	2.00	30.55	760.80
36.0	4.00	31.09	816.72	3.00	32.32	864.94
39.0	4.00	34.09	975.75	3.00	35.32	1028.38

Table 14-6 SAE Specifications and Strengths for Steel Bolts

SAE Grade Number	Size Range Outside Diameter (in)	Minimum Proof Strength (kpsi)	Minimum Yield Strength (kpsi)	Minimum Tensile Strength (kpsi)	Material
1	0.25–1.5	33	36	60	low or medium carbon
2	0.25–0.75	55	57	74	low or medium carbon
2	0.875–1.5	33	36	60	low or medium carbon
4	0.25–1.5	65	100	115	medium carbon, cold drawn
5	0.25–1.0	85	92	120	medium carbon, Q&T*
5	1.125–1.5	74	81	105	medium carbon, Q&T
5.2	0.25–1.0	85	92	120	low-carbon martensite, Q&T
7	0.25–1.5	105	115	133	medium-carbon alloy, Q&T*
8	0.25–1.5	120	130	150	medium-carbon alloy, Q&T
8.2	0.25–1.0	120	130	150	low-carbon martensite, Q&T

* Quenched and Tempered.

Table 14-7 Metric Specifications and Strengths for Steel Bolts

ISO Class Number	Size Range Outside Diameter (mm)	Minimum Proof Strength (MPa)	Minimum Yield Strength (MPa)	Minimum Tensile Strength (MPa)	Material
4.6	M5–M36	225	240	400	low or medium carbon
4.8	M1.6–M16	310	340	420	low or medium carbon
5.8	M5–M24	380	420	520	low or medium carbon
8.8	M3–M36	600	660	830	medium carbon, Q&T
9.8	M1.6–M16	650	720	900	medium carbon, Q&T
10.9	M5–M36	830	940	1 040	low-carbon martensite, Q&T
12.9	M1.6–M36	970	1 100	1 220	alloy, quenched & tempered

steels:
$$\begin{cases} S_{e'} \cong 0.5\,S_{ut} & \text{for } S_{ut} < 200 \text{ ksi } (1\,400 \text{ MPa}) \\ S_{e'} \cong 100 \text{ ksi } (700 \text{ MPa}) & \text{for } S_{ut} \ge 200 \text{ ksi } (1\,400 \text{ MPa}) \end{cases}$$

irons:
$$\begin{cases} S_{e'} \cong 0.4\,S_{ut} & \text{for } S_{ut} < 60 \text{ ksi } (400 \text{ MPa}) \\ S_{e'} \cong 24 \text{ ksi } (160 \text{ MPa}) & \text{for } S_{ut} \ge 60 \text{ ksi } (400 \text{ MPa}) \end{cases}$$

aluminums:
$$\begin{cases} S_{f_{@5E8}} \cong 0.4\,S_{ut} & \text{for } S_{ut} < 48 \text{ ksi } (330 \text{ MPa}) \\ S_{f_{@5E8}} \cong 19 \text{ ksi } (130 \text{ MPa}) & \text{for } S_{ut} \ge 48 \text{ ksi } (330 \text{ MPa}) \end{cases}$$

copper alloys:
$$\begin{cases} S_{f_{@5E8}} \cong 0.4\,S_{ut} & \text{for } S_{ut} < 40 \text{ ksi } (280 \text{ MPa}) \\ S_{f_{@5E8}} \cong 14 \text{ ksi } (100 \text{ MPa}) & \text{for } S_{ut} \ge 40 \text{ ksi } (280 \text{ MPa}) \end{cases}$$

Steel: $S_e' = 0.45 S_{ut}$

The following are standard thread lengths:

$$l_{thd} = \begin{cases} 2d + \tfrac{1}{4} \text{ in} & L \le 6 \text{ in} \\ 2d + \tfrac{1}{2} \text{ in} & L > 6 \text{ in} \end{cases}$$

$$l_{thd} = \begin{cases} 2d + 6 \text{ mm} & L \le 125 \text{ mm} \quad d \le 48 \text{ mm} \\ 2d + 12 \text{ mm} & 125 < L \le 200 \text{ mm} \\ 2d + 25 \text{ mm} & L > 200 \text{ mm} \end{cases}$$

For a round bolt of diameter d and axially loaded thread length l_t within its clamped zone of length l as shown in Figure 14-21, the spring constant is

$$\frac{1}{k_b} = \frac{l_t}{A_t E_b} + \frac{l - l_t}{A_b E_b} = \frac{l_t}{A_t E_b} + \frac{l_s}{A_b E_b} \tag{14.11a}$$

where A_b is the total cross-sectional area and A_t is the tensile-stress area of the bolt, and $l_s = (l - l_t)$ is the length of the unthreaded shank. The length of the threaded portion is standardized as twice the bolt diameter plus 1/4 in for U.S. bolts (plus 6 mm for metric bolts) up to 6 in (150 mm) long. An additional 1/4 in of thread is provided on longer bolts. Bolts shorter than the standard thread length are threaded as close to the head as possible.[2]

For the cylindrical material geometry in Figure 14-22 (ignoring the flanges) the material spring constant becomes

$$\frac{1}{k_m} = \frac{l_1}{A_{m_1} E_1} + \frac{l_2}{A_{m_2} E_2} = \frac{4 l_1}{\pi D_{eff_1}^2 E_1} + \frac{4 l_2}{\pi D_{eff_2}^2 E_2} \tag{14.11b}$$

where the A_m are the effective areas of the clamped materials and the D_{eff} are the effective diameters of those areas.

If both clamped materials are the same

$$k_m = \frac{A_m E_m}{l} \tag{14.11c}$$

where A_m is the effective area of the clamped material (also see Section 14.8). If A_m can be defined as a solid cylinder with an effective diameter D_{eff}, equation 14.11c becomes

$$k_m = \frac{\pi D_{eff}^2}{4} \frac{E_m}{l} \tag{14.11d}$$

$$P = P_m + P_b$$

The compressive load F_m in the material is now

$$F_m = F_i - P_m; \qquad F_m \ge 0^*$$

and the tensile load F_b in the bolt becomes

$$F_b = F_i + P_b$$

$$A_m = \frac{\pi}{4}\left[\left(\frac{d_2 + d_3}{2}\right)^2 - d^2\right] \tag{14.17a}$$

The material stiffness k_m is then found from equation 14.11 (p. 903) using diameters d, d_2, d_3 and length $l = l_m$ as defined in Figure 14-31 with $\phi = 30°$.

A more extensive study of joint stiffness using FEA was done by Wileman et al.[13] who fitted an empirical equation to their extensive finite-element-modeled data that de fines an approximate material stiffness parameter k_m as a function only of bolt diame ter d, clamped length l, and the clamped material's modulus of elasticity E. It works for any units system because the exponential expression is dimensionless.

$$k_m = d\,E\,A\,e^{b(d/l_m)} \tag{14.17b}$$

The value of k_m is used in equation 14.13c (p. 905) to find the *joint stiffness constant* C. The coefficient A and exponent b in equation 14.17b will vary slightly with the Pois son ratio ν of the material. Table 14-9 shows these equation parameters for several com mon metals. For materials not shown, use the coefficients from the table for a Poisson ratio that is closest to that of the material used, or use the general expression. Wileman

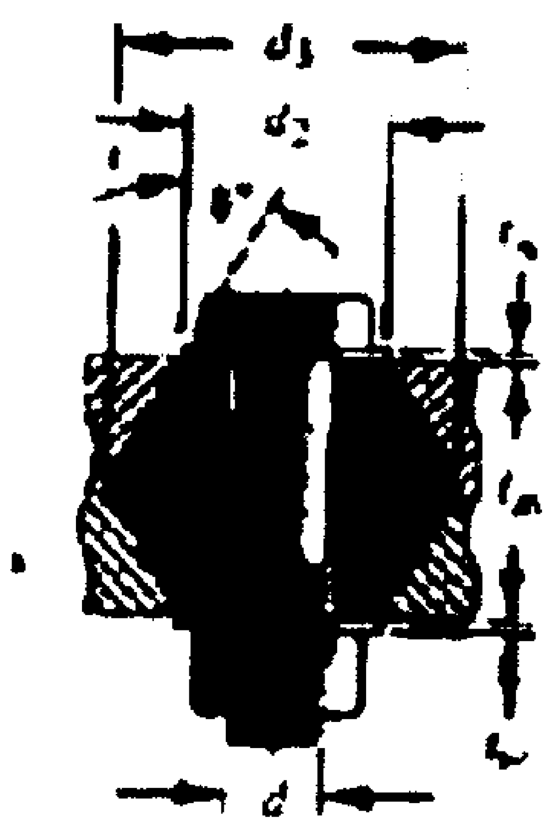
Table 14-9 Stiffness Parameters for Equation 14.17 [13]

Material	ν	E (GPa)	A	b
Steel	0.291	206.8	0.78715	0.62873
Aluminum	0.334	71.0	0.79670	0.63816
Copper	0.326	118.6	0.79568	0.63553
Gray Cast Iron	0.211	100.0	0.77871	0.61616
General Expression (averaging all four tested materials)			0.78952	0.62914

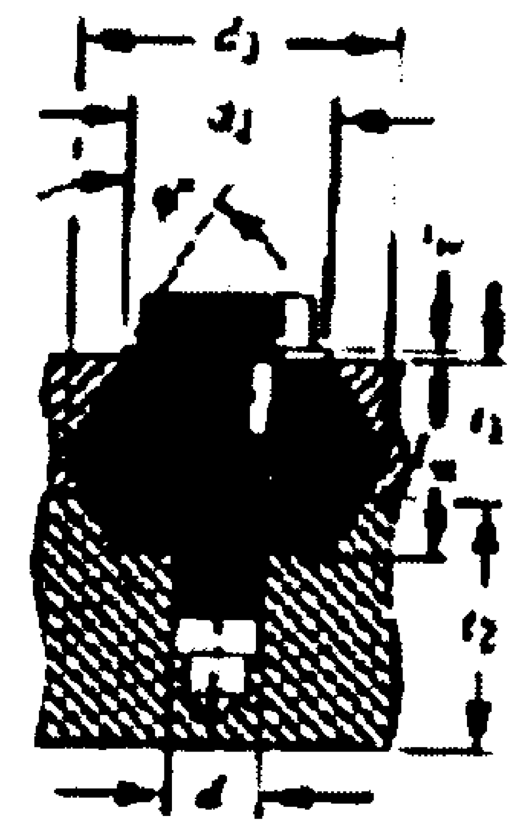
$T_i \cong K_i F_i d$ $\qquad$ $T_i \cong 0.21 F_i d$

Table 14-11
Torque Coefficient K_i for UNS Standard Threads with Coefficients of Friction $\mu = \mu_c = 0.15$

Bolt Size	K_i UNC	K_i UNF
0		0.22
1	0.22	0.22
2	0.22	0.22
3	0.22	0.22
4	0.22	0.22
5	0.22	0.22
6	0.22	0.22
8	0.22	0.22
10	0.22	0.21
12	0.22	0.22
1/4	0.22	0.21
5/16	0.22	0.21
3/8	0.22	0.21
7/16	0.21	0.21
1/2	0.21	0.21
9/16	0.21	0.21
5/8	0.21	0.21
3/4	0.21	0.21
7/8	0.21	0.21
1	0.21	0.21
1 1/8	0.21	0.21
1 1/4	0.21	0.21
1 3/8	0.21	0.21
1 1/2	0.21	0.20
1 3/4	0.21	
2	0.21	
2 1/4	0.21	
2 1/2	0.21	
2 3/4	0.21	
3	0.21	
3 1/4	0.21	
3 1/2	0.21	
3 3/4	0.21	
4	0.21	



$l_p = l_m + 2t_w$
$d_2 = 1.5d$
$d_3 = 1.5d + l_m \tan\phi$
(a) Bolt-frusta model



$l_m = t_1 - t_2/2$ if $t_2 < d$
$l_m = t_1 + d/2$ if $t_2 \ge d$
$l_p = l_m + t_w$
$d_2 = 1.5d$
$d_3 = 1.5d + l_m \tan\phi$
(b) Cap screw frusta

FIGURE 14-31
Estimating the Material Compressed by a Bolt or Cap Screw Using the Cone Frusta Method[12]

We can summarize the information in Figure 14-24 in the following way. The common change in deflection $\Delta\delta$ due to the applied load P is

$$\Delta\delta = \frac{P_b}{k_b} = \frac{P_m}{k_m} \qquad (14.13a)$$

or:

$$P_b = \frac{k_b}{k_m} P_m \qquad (14.13b)$$

Substitute equation 14.12a to get

$$P_b = \frac{k_b}{k_m + k_b} P$$

or

$$P_b = CP \qquad \text{where} \quad C = \frac{k_b}{k_m + k_b} \qquad (14.13c)$$

The term C is called the joint's *stiffness constant* or just the joint constant. Note that C is typically < 1, and if k_b is small compared to k_m, C will be a small fraction. This confirms that the bolt will see only a portion of the applied load P.

In like fashion,

$$P_m = \frac{k_m}{k_b + k_m} P = (1 - C)P \qquad (14.13d)$$

These expressions for P_b and P_m can be substituted into equations 14.12b and 14.12c to get expressions for the bolt and material loads in terms of the applied load P:

$$F_m = F_i - (1 - C)P \qquad (14.14a)$$

$$F_b = F_i + CP \qquad (14.14b)$$

Equation 14.14b can be solved for the preload F_i needed for any given combination of applied load P and maximum allowable bolt (proof) load F_b, provided that the joint constant C is known.

The load P_0 required to separate the joint can be found from equation 14.14a by setting F_m to zero.

$$P_0 = \frac{F_i}{(1 - C)} \qquad (14.14c)$$

A safety factor against joint separation can be found from

$$N_{separation} = \frac{P_0}{P} = \frac{F_i}{P(1 - C)}$$

will be demonstrated in an example. The fatigue safety factor can be calculated without drawing the Goodman diagram by employing equation 13.34b (p. 851), using notation consistent with this section.

$$N_f = \frac{S_e(S_{ut} - \sigma_i)}{S_e(\sigma_m - \sigma_i) + S_{ut}\sigma_a} \qquad (14.16)$$

$$n = \frac{S_y}{\sigma_{max}} = \frac{S_y}{\sigma_m + \sigma_a}$$

$$F_{alt} = \frac{F_b - F_i}{2}, \qquad F_{mean} = \frac{F_b + F_i}{2}$$

where F_b is found from equation 14.14b (p. 905) with $P = P_{max}$.

The mean and alternating stresses in the bolt are

$$\sigma_a = K_f \frac{F_{alt}}{A_t}, \qquad \sigma_m = K_{fm} \frac{F_{mean}}{A_t}$$

Table 14-8 Fatigue Stress-Concentration Factors for Bolts

Brinell Hardness	SAE Grade (UNS)	Metric Class (ISO)	K_f Rolled Threads	K_f Cut Threads	K_f Fillet
< 200 (annealed)	≤ 2	≤ 5.8	2.2	2.8	2.1
> 200 (hardened)	≥ 4	≥ 6.6	3.0	3.8	2.3

The stress due to the preload force F_i is

$$\sigma_i = K_{fm} \frac{F_i}{A_t} \qquad (14.15c)$$

if $K_f|\sigma_{max_{nom}}| < S_y$ then: $\quad K_{fm} = K_f$

if $K_f|\sigma_{max_{nom}}| > S_y$ then: $\quad K_{fm} = \dfrac{S_y - K_f\sigma_{a_{nom}}}{|\sigma_{m_{nom}}|} \qquad (6.17)$

if $K_f|\sigma_{max_{nom}} - \sigma_{min_{nom}}| > 2S_y$ then: $\quad K_{fm} = 0$

Fully Corrected Endurance Limits for Bolts and Screws with Rolled Threads

GRADE OR CLASS	SIZE RANGE	ENDURANCE LIMIT
SAE 5	$\frac{1}{4}$–1 in	18.6 kpsi
	$1\frac{1}{8}$–$1\frac{1}{2}$ in	16.3 kpsi
SAE 7	$\frac{1}{4}$–$1\frac{1}{2}$ in	20.6 kpsi
SAE 8	$\frac{1}{4}$–$1\frac{1}{2}$ in	23.2 kpsi
ISO 8.8	M16–M36	129 MPa
ISO 9.8	M1.6–M16	140 MPa
ISO 10.9	M5–M36	162 MPa
ISO 12.9	M1.6–M36	190 MPa