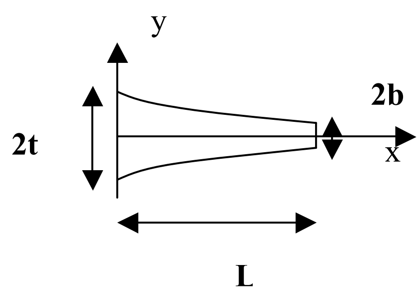
HEAT TRANSFER CANDIDACY EXAM - SPRING 2007

1. Consider a longitudinal thin fin of concave parabolic profile ($y = (b-t)\, x^2/L^2 + t$) exposed to an ambient temperature of T_∞ and a convection coefficient h. The fin base at x=0 is at a temperature of T_b and the fin tip at x=L is at a temperature of T_L.



(i) By considering an energy balance on an appropriate control volume, find the governing equation for the temperature T(x).

(ii) Solve for the temperature variation T(x).

(iii) Write an expression for the heat removal rate from the fin. Determine this for T(x) in part (ii).

Note: the solution to the following differential equation:

$$t^2 \frac{d^2u}{dt^2} + kt\frac{du}{dt} + pu = 0$$

is: $u(t) = c_1\, t^{m1} + c_2\, t^{m2}$

Where m1 and m2 are two distinct roots of:

$m(m-1) + km + p = 0$

2. The figure shows a stationary bottom flat plate and a top flat plate that is moving at constant velocity, U, with respect to the bottom plate. A thin layer of fluid separates the plates. The separation distance is L and the temperatures of the top and bottom plates are constant and equal to T_1 and T_0, respectively. The liquid contained between the two plates has constant properties (μ, ρ, C_p, and k) and the induced flow is assumed to be laminar and fully developed. Neglecting body forces, pressure gradients, and assuming steady one-dimensional flow,

1. Develop a relationship to predict the temperature profile in the fluid, i.e. $T = T(y)$
2. Given the definitions of the Prandtl number , Pr, and Eckert numbers, E, develop a relationship between the Nusselt number, Nu, and these non-dimensional numbers for this problem.

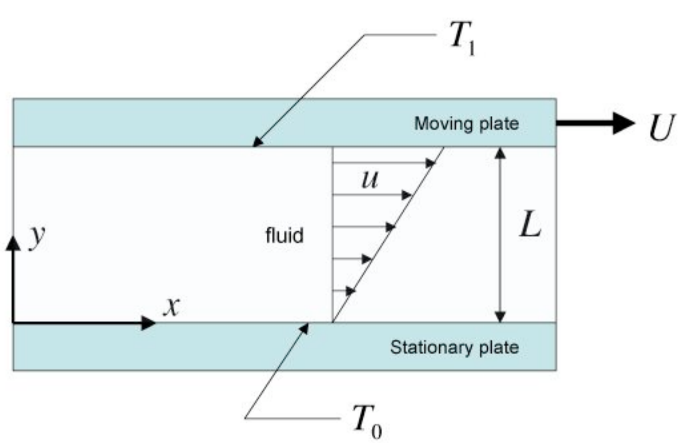
Given:

$$\Pr = \frac{C_p \mu}{k}, \quad E = \frac{U^2}{\mathrm{C_p}\left(T_1 - T_0\right)}, \quad Nu = \frac{hL}{k}, \text{ and where, } h = \frac{k\left(\dfrac{\partial T}{\partial y}\right)_{y=0}}{T_1 - T_0}$$

The appropriate form of the energy equation is: $\rho C_p \dfrac{DT}{Dt} = \nabla \bullet \left(k \nabla T\right) + \mu \phi$

Where: $\dfrac{D(\ )}{Dt} = \dfrac{\partial(\ )}{\partial t} + \bar{\mathrm{v}} \bullet \nabla(\ ), \quad \bar{\mathrm{v}} = u\bar{i} + v\bar{j} + w\bar{k}, \quad \nabla = \bar{i}\dfrac{\partial(\ )}{\partial x} + \bar{j}\dfrac{\partial(\ )}{\partial y} + \bar{k}\dfrac{\partial(\ )}{\partial z}$

$$\phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2$$



3. Consider the radiation exchange with a 1mm diameter ceramic sphere, highly polished with thermal conductivity 5 W/mK, and spectral emissivity indicated bellow.

$\varepsilon_\lambda = 0.6\ \lambda < 3\mu m$ $\qquad\qquad$ $\varepsilon_\lambda = 0.2\ \lambda > 3\mu m$

The surface heat transfer coefficient is $15 W/m^2K$ and the sphere is irradiated by sunlight at irradiation of $1400\ W/m^2$ with normal solar spectrum ($T_{sun} = 5800K$). Temperature of the surroundings take as 300K.

(a) Indicate equation for energy balance, and indicate equation from which the transient temperature would be calculated. List assumptions that you make.
(b) Calculate the **steady-state temperature** of the center of the sphere and the surface temperature of the sphere.
(c) Calculate the **steady-state temperature** of the sphere when there is a surface reaction with the air taking place which generated a constant surface heat flux of $10^5\ W/m^2$.

Blackbody Radiation Functions

λT (μmK)	F(0-λ)
1000	0.00032
1200	0.00213
1400	0.00779
1600	0.01972
1800	0.03934
2000	0.06673
2200	0.10089
2400	0.14026
2600	0.18312
2800	0.22790
3000	0.27323
3200	0.31810
3400	0.36174
3600	0.40361
3800	0.44338
4000	0.481
4200	0.516
4400	0.549
4600	0.579
4800	0.608
5000	0.633
5200	0.659

D=diameter $\qquad$ Area of sphere = πD^2 $\qquad$ Volume of sphere = $(\pi D^3)/6$