1. Conduction Question.

A metallic heater is attached to a slab of material, as seen in figure below. The contact resistance at the interface may be neglected and both slabs are very long and deep (in the plane perpendicular to the paper).



Heat is generated at a uniform volumetric rate q''' in the heater starting at time t = 0, when both slabs are at a temperature T_i . The left boundary of the heater and the right boundary of the slab may be considered adiabatic at all times.

- A) Write the governing equations and the necessary conditions to determine the temperature in the two slabs as a function of time. (20%)B)
- B) What is the condition that allows the neglect of the spatial variation of temperature within the heater ? What is the resulting equation for the heater temperature variation with time ? (20%)
- C) What is the condition that allows the neglect of the thermal storage within the heater ? What is the resulting condition at the interface of the heater and the material slab ? (15%)
- D) Show the expected temperature variation across the slab at two times for the conditions of Part 3. Will there be a steady state in this problem ? Explain why or why not. In case of a steady state, show the temperature profile in steady state. (15%)
- E) Solve for the transient temperature variation in the slab for conditions in Part 3. Any unknown constants may be left undetermined, with a brief explanation of how they can be solved for. (30%)

2. Convection Question

Consider a long **horizontal** pipe of diameter 3cm at a constant temperature 100° C in a water bath at a temperature of 20C



A correlation often used for a heated cylindrical pipe is the Churchill Chu:

$$\overline{Nu_D} = \left\{ 0.60 + \frac{0.387Ra_D^{\frac{1}{6}}}{\left[1 + (0.559/\Pr)^{\frac{9}{16}}\right]^{\frac{8}{27}}} \right\}^2 \qquad Ra_L = Gr_L \Pr = \frac{g\beta(T_s - T_{\infty})L^3}{v\alpha}$$

A) Estimate the Ra number and Nu number, given the following properties for water: $\rho = 1000 \text{ kg/m}^3$, $c_p = 400 \text{ J/kgK}$, k = 0.7 W/mK, $\alpha = 3x10^{-7} \text{ m}^2/\text{s}$, $\beta = 7.5x10^{-4} \text{ K}^{-1}$, and Pr = 1.8.

B) Sketch the velocity and temperature next the cylinder as a function of radial distance. Indicate the approximate numerical value of the boundary layer thickness.

C) Sketch in a separate diagram how the temperature and velocity vary at other angles of theta, including theta =0, 90 and 180° . (Theta is angle with respect to gravitational field) Assume the flow is stationary other than flow induced by buoyancy forces.

D) If the water in the bath is moving with velocity U, estimate the velocity at which forced convection would begin to play a significant role in defining the steady-state heat transfer from the pipe.

E) Discuss qualitatively how the velocity and temperature gradient would differ if the pipe temperature was 200°C. What other factors should be considered to find the total heat loss from the pipe under these conditions.

3. Radiation Question

An oven used to anneal a coating on the surface of metal panels is fabricated by creating a triangular enclosure with heaters placed inside one of the walls. The second wall has some insulation, which does not totally prevent heat loss through the wall. The third wall is made from the metal panel with the coating needing to be annealed. The cross section of the oven forms an equilateral triangle with each side measuring 1m in length. Thermocouples readings show steady state surface temperatures inside the oven as given in the diagram. It is desired to analyze the radiation exchange in this system.



A) To determine the total emissivity of coating on the metal panel, a colleague suggests that a sample of the coating be heated to 300°C and a photodetector be used to measure the normal absorptivity after illumination with a visible laser. The coating can be considered opaque and gray and test results determine that $\alpha_n = 0.5$. Is this experiment sufficient or what assumptions need to be made to determine the total emissivity.

B) An IR detector needs to be selected to measure the panel temperature during the absorptivity experiment. One has a maximum sensitivity in the 3-5 μ m wavelength regime and 20% quantum efficiency, the other with a max sensitivity between 10-14 μ m and a 50% quantum efficiency. Which detector would be better to use?

C) Assuming that the conditions needed in part A are met such that $\varepsilon_{Total} = 0.5$, determine the heat per unit length required by the heaters and the cooling per unit length that must be supplied to the panel to maintain this furnace setup.

D) If the insulation on surface 2 is replaced such that it becomes a perfect insulator, what changes about the problem and how would you analyze it. What is the power required by the heaters and the cooling requirement for the panel?