Beam Equations

$$\sigma_{xx}(x,y) = \frac{-M(x)y}{I_{zz}} \qquad \sigma_{xy} = \frac{V(x)Q_p(y)}{I_{zz}b}$$

$$\frac{dV}{dx} = -q(x) \qquad \frac{dM}{dx} = V(x) \qquad EI_{zz}\frac{d^2v}{dx^2} = M(x)$$

First Moment of Area of a rectangular cross section (above point p)

$$Q_p = \int y\,dA_p = \int_{-b/2}^{b/2}\int_{y}^{h/2} y\,dy\,dz = \frac{b}{2}\left(\frac{h^2}{4} - y\right)$$

Second Moment of Area of a rectangular cross section

$$I_{zz} = \frac{bh^3}{12}$$

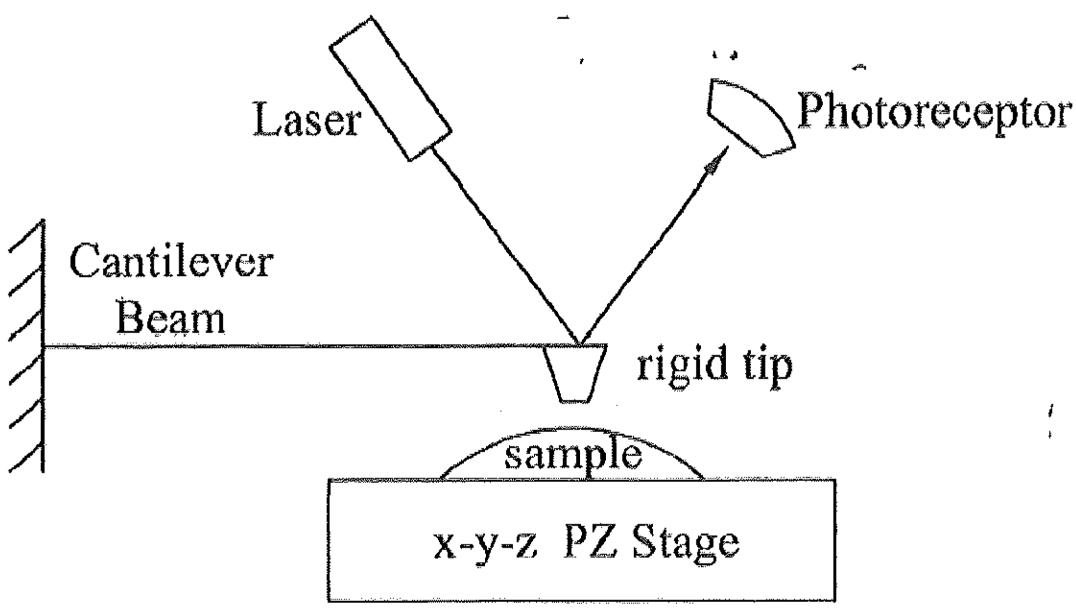
1. The atomic force microscope (AFM), first reported in 1986, has become a widely used tool to study forces between and within individual molecules on the order of picoNewtons (10-12 pN) and the topology of surfaces, including cells, with resolution on the order of nanometers. A schematic of the basic design of an AFM probe is shown below. It consists of a cantilever beam with a rigid end tip. The laser and photodetector are used to measure changes in the angle of the laser light (i.e., the end slope $\phi = \frac{dv}{dx}(x = L)$) that are associated with the deflection of the cantilever. If the cantilever beam (which has a length L, a second moment of area I_{zz}, and a Young's modulus E) is lowered onto the soft sample, the sample will impose a force P on the cantilever tip. Given that we can measure the end slope ϕ of the cantilever, please answer the following.

(a) Let $\delta = v(x = L)$ be the deflection at the end of the beam. What is the deflection δ in terms of the material parameters, geometry, and measured values? Note: this result should NOT include the unknown applied load P.

(b) What is the force P in terms of material parameters, geometry, and measured values?

(c) The effective stiffness, k, of the cantilever beam is defined through the relation $P = k\delta$. What is the value of k in terms of material parameters and geometry? Note, given the similarity between $P = k\delta$ and the classical force displacement relation for a spring, $f = \hat{k}\hat{\delta}$, k is often called the AFM spring constant.

(d) If L=400 μm and the beam is rectangular and made of silicon (E=166 GPa) and if the width of the beam $b = 5h$, where h is the height of the beam, what value of h will yield an effective stiffness of k=1.0 N/m?



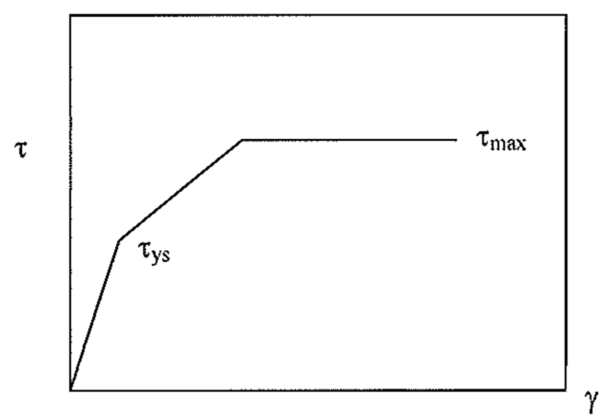
2. In parts (a)-(d), consider two long shafts of circular cross section, the first shaft being hollow and the second shaft being solid. The ratio of the inside to outside diameter of the first shaft is 7/8. Both shafts are made from the same material. Neglect end effects. The material yield strength in shear is τ_{ys}

   (a) Determine the diameter of the second (solid) shaft in terms of the outside diameter of the first shaft such that the twist per unit length for a given applied torque is the same for both shafts in the range of linear elastic behavior. State all assumptions and requirements of your solution.

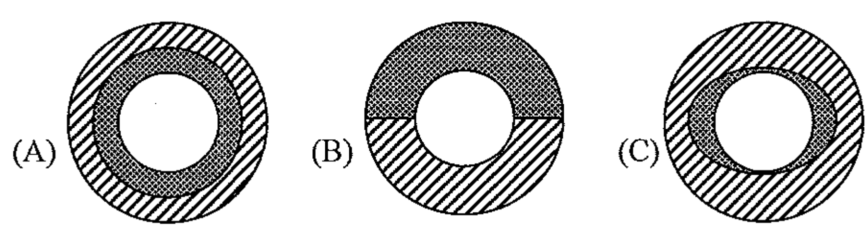
   (b) What is the ratio of torque applied to the first shaft to that of the second shaft with dimensions determined in part (a) at the point of initial yielding in each shaft?

   (c) How does the solution in part (b) change if each shaft is subjected to the same value of superimposed bending moment, M, assuming a von Mises yield function? Again, use dimensions of the second shaft determined in part (a).

   (d) If the material exhibits a trilinear shear stress-strain relation shown below, with a plateau of maximum shear stress at the value τ_{max}, please derive the ratio of the maximum torque (limit torque) of the first shaft to that of the second shaft. Assume that pure torque is applied (no bending moment) and use dimensions of the second shaft determined in part (a).

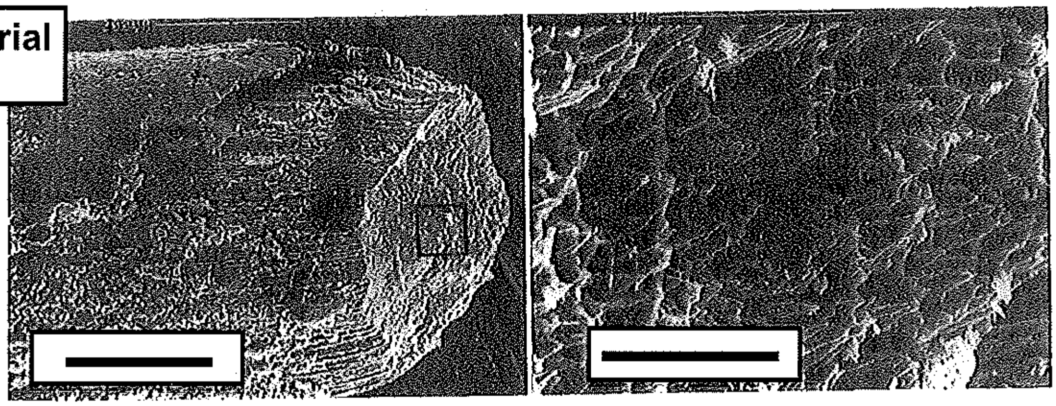


   (e) Do the equations of simple torsion theory of elastic shafts from strength of materials apply for each bimaterial shaft cross section shown below? Please explain your answer thoroughly for each of (A), (B) and (C) below.



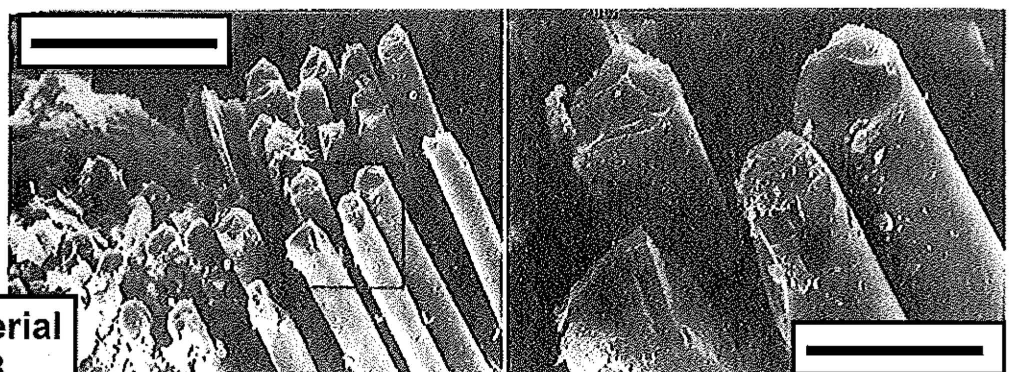
3. The pictures in the following figures show the failure surface of three different materials.
   1) What kind of failure has each material undergone?
   2) For each case, show the stress-strain curves (engineering and true) for the loading conditions from zero to failure;
   3) For each case, give an example of a material and a load that would cause this kind of failure.

Material A



The bars represent 200 and 20microns, respectively.

Material B



The bars represent 100 and 20microns, respectively.

Material C