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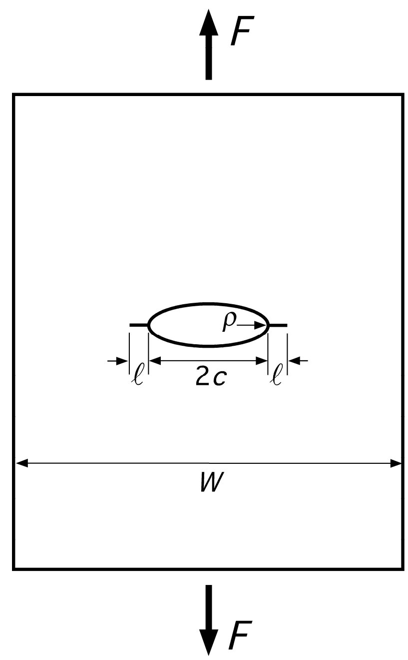
Mechanics of Materials
PhD Qualifying Examination
Spring 2006
Please work all four problems – 25 points each (show ALL work)

1. The plate contains an elliptical hole with cracks of length ℓ at both ends of the major axis and is loaded as shown. In addition to the parameters given in the figure, you are given the thickness, t, fracture toughness, K_c, and yield strength, σ_o. The material is elastic-perfectly plastic, and the elastic stress concentration factor for the elliptical hole (i.e., without cracks) is given by



$$k_t = 1 + 2\sqrt{\frac{c}{\rho}} \;,$$

where ρ is the radius of curvature at the location shown in the figure.

(a) Estimate the fracture load, F_f, for the following three cases in terms of parameters given. State any necessary assumptions.

(i) Brittle behavior with $\ell / c = 0.01$ and $W >> 2c$

(ii) Brittle behavior with $\ell / c = 1$ and $W >> 2c$

(iii) Ductile behavior with $\ell / c = 0.01$ and $W >> 2c$

(b) For a given load, F, what is the crack length, ℓ, at fully plastic yielding?

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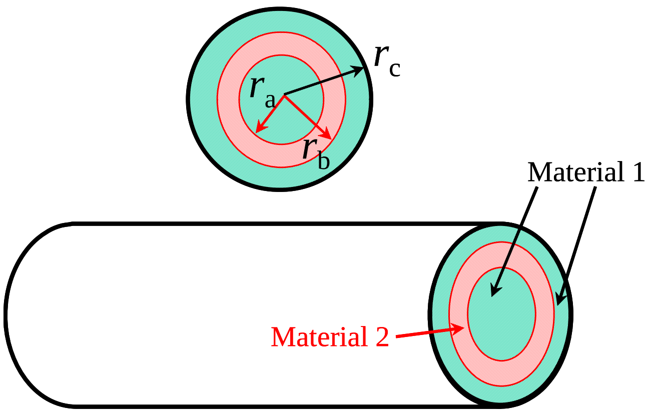
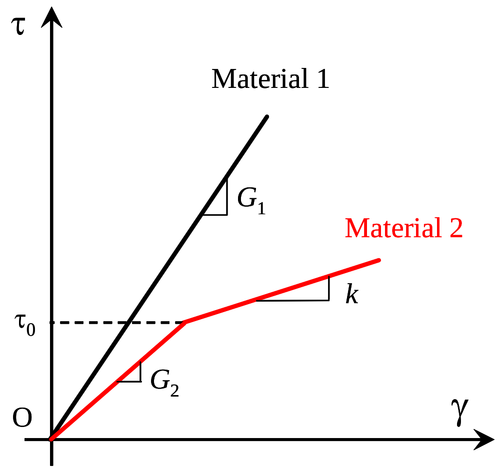
2. A shaft made of two different materials is designed to transmit torque applied at its two ends. The cross section consists of an inner core, an outer ring, and an annular infill. The inner core and the outer ring are made of material 1 while the infill is made of material 2. The shear stress-strain curves for the two materials are shown in the figure, with τ_0 being the yield strength in shear for material 2. Assume the two materials are perfectly bonded at the interfaces and the engineering approach of analysis can be applied here.





(1) Provide a qualitative graphical illustration of the distributions of stress on the cross section as a function of applied torque. State the assumptions behind your analysis. You do not need to provide closed form expressions as long as you illustrate the key attributes of the solution.

(2) For $G_1 = G_2$, find the minimum torque required to cause yielding at all points in the infill (complete yielding of the infill).

(3) For $G_1 = G_2$, if the torque is gradually increased to the minimum value necessary to achieve complete yielding of the infill and then decreased to zero, calculate the residual stress at the outer edge $r = r_c$.

(4) Under what conditions are the stress distributions you illustrated in part (a) representative of the fully accurate solution that considers interfaces and distinct properties of phases? Illustrate qualitatively the characteristics you expect from the fully accurate solution by indicating how the accurate solution might be qualitatively different from the solution you gave in part (a).

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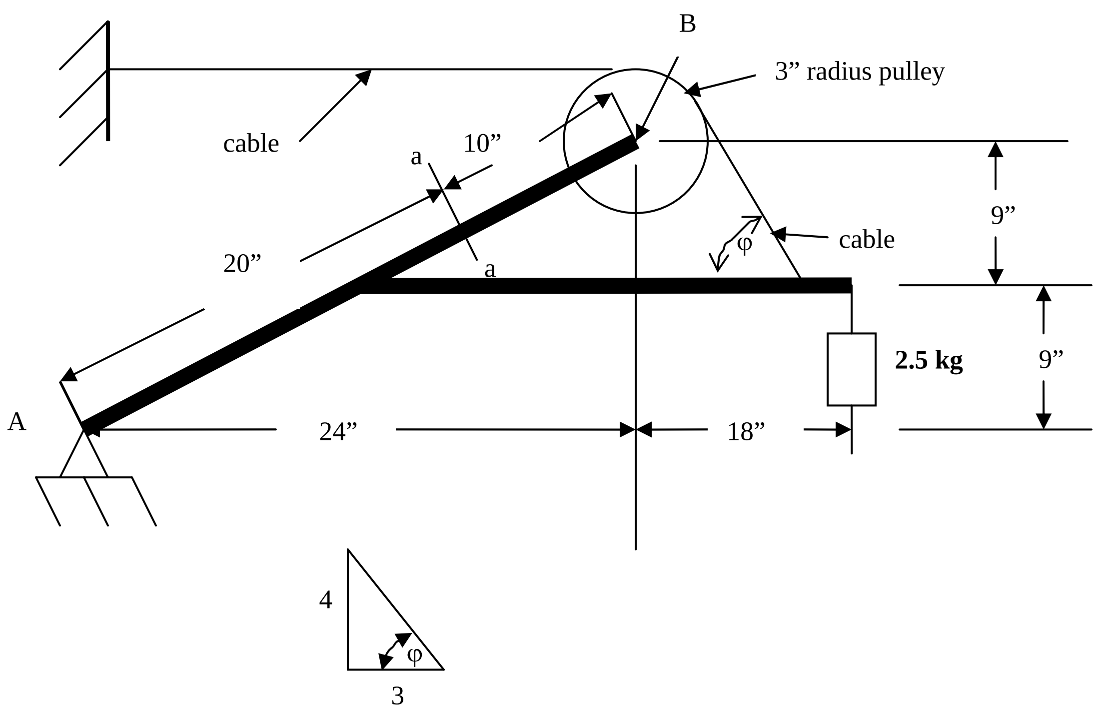
3. Shown below is a structure loaded by a 2.5 kg weight hung from a cable as shown. The structure is pinned at point A and has a pulley connection at point B.

a. Determine the shear force, bending moment and axial force at section a-a.
b. If the inclined member AB has a 3”x5” rectangular cross section (3” height, 5” width), determine the maximum stresses at section a–a due to bending, axial force and shear force.
c. Show the directions of the stresses computed on appropriately labeled elements.

Clearly state all assumptions.



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4. Consider the square cross-section beam shown below, of dimension h by h, subject to "diamond-orientation" bending with neutral axis in the plane y = 0. The beam is composed of an isotropic, elastic/perfectly-plastic material having Young's modulus E and yield strength σ_y, and is subjected to pure bending moment, M.

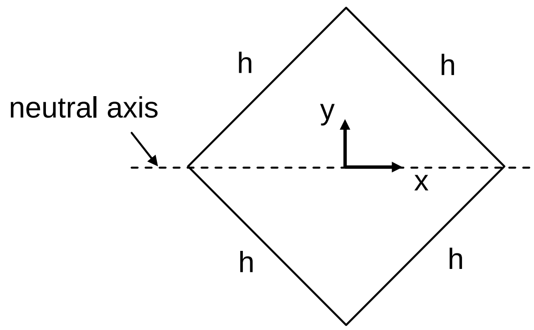

Fig.1 Square cross-section of beam, oriented for bending along the "diamond" orientation.

1. Using the standard assumptions of beam theory, evaluate the magnitude of applied bending moment, $\mathrm{M_i}$, just sufficient to bring the most highly-stressed region to the verge of yielding.

2. If the applied moment is increased to very large values, the elastic/plastic boundaries (tension and compression sides) in this geometry will move inward, toward the neutral axis. At an even larger applied moment, the boundaries will reach the y = 0 neutral axis, resulting in tensile yielding stress values of magnitude σ_y in one triangular half of the cross-section, and compressive yielding stress values of magnitude $-\sigma_y$ in the other triangular half of the cross-section. At this point, the bending moment carried by the cross-section reaches a limiting value, $\mathrm{M_L}$. Evaluate $\mathrm{M_L}$.

3. Using your answers to the two previous questions, evaluate the ratio $\mathrm{M_L}/\mathrm{M_i}$ for bending of this beam. How does this value compare with the ratio for bending of this same cross-section, but with the neutral axis rotated 45 degrees from that shown in the above figure?

4. Compare $\mathrm{M_i}$ for the "diamond" cross-section with the corresponding $\mathrm{M_i}$ for the square orientation. What is the ratio of these first-yield bending moments? Explain why they differ. Evaluate the same ratio for the corresponding limit moments, and comment on reasons why they differ.