Problem 1

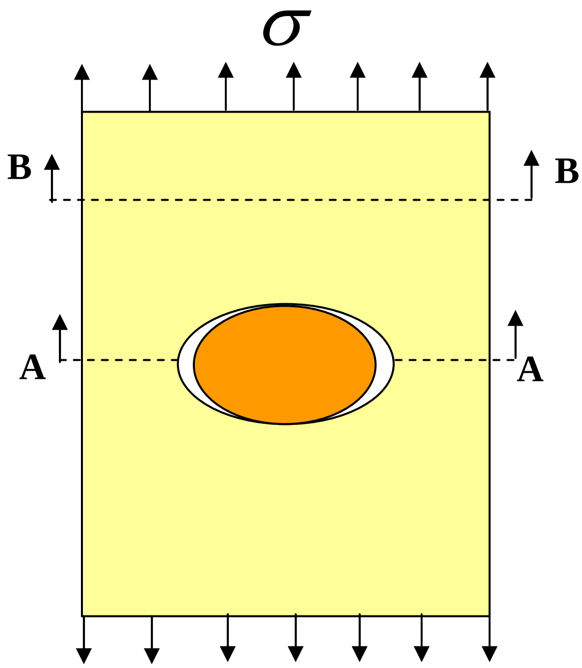

Figure 1-1

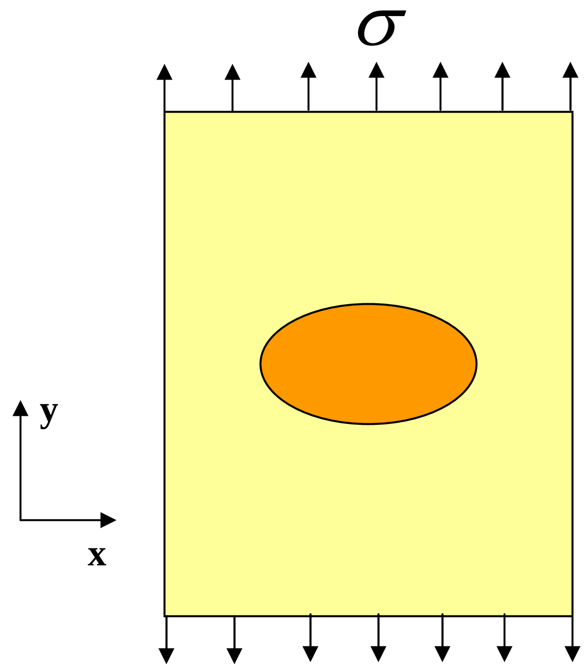

Figure 1-2

A plate is loaded by a remote uniform stress field in tension as shown in the figures above. Before the load is applied, the plate has an ellipsoidal hole that either (i) has an ellipsoidal inclusion that just contacts the top and bottom of the hole (Figure 1-1) or (ii) is completely filled by an ellipsoidal inclusion with a perfect fit (Figure 1-2). Assume for parts a-e below that the inclusion material is much stiffer than the plate material and both are homogeneous, isotropic and linear elastic.

a. Assuming that there is no bonding of the interface between inclusion and the plate, provide a qualitative sketch of the remote (plate) stress-strain curve in each case. Write a general form of the elastic strain energy of the system for each case, being as precise as possible.

b. For the case at upper left, please qualitatively sketch σ_{yy} as a function of x along both of the sections A-A and B-B (far from hole) for the case in Figure 1-1. What principle allows us to neglect stress concentration at cross sections that are far removed from the hole?

c. Is the theoretical elastic stress concentration factor, K_t, for a plate with an elliptical hole but no inclusion relevant to solving for the stress concentration in either or both of the two cases shown above?

d. What specific material properties are relevant to solving the problem in Figure 1-2? Explain.

e. If the inclusion is perfectly bonded to the matrix in the problem in Figure 1-2, please draw a qualitative sketch of remote stress-strain behavior (e.g. section B-B) and compare with the remote stress-strain curve for the same geometry for the case when the inclusion has a frictionless, unbonded interface. Are they different? Explain.

f. If the inclusion stiffness is much, much lower than that of the plate, revisit questions c, d and e above. Assume both plate and inclusion are homogeneous, isotropic and linear elastic.

g. Now suppose that the plate material can undergo plastic flow at a yield strength σ_y (see Figure 1-3), while the inclusion remains linear elastic. Assume the plate material is elastic-perfectly plastic (see below). If the inclusion has the same stiffness as the matrix and is perfectly bonded for the upper right case, draw the remote stress-strain curve for each plate shown above. Is the fully plastic limit load the same for both cases? Explain.

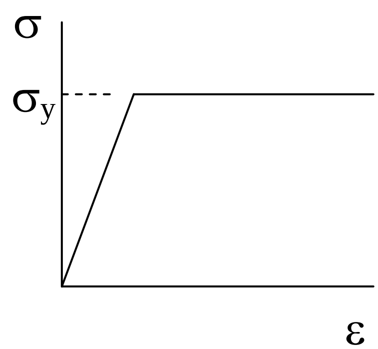

Figure 1-3

Problem 2

A microscale cantilever beam is fabricated by perfectly bonding three thin and slender metallic strips together as shown in Figure 2-1. An accidental overheating incident occurred during an operation. After the cantilever is retrieved and cooled to ambient temperature, it is found that the beam is slightly longer than before with an elongation of Δ at the free end. The thermal expansion coefficients of the two materials are shown. Furthermore, the temperature-independent stress-strain curves of these materials are given in Figure 2-2. If any other information you need to answer the questions below is not given, you are free to make assumptions, as long as you clearly state what the assumptions are.

1) Please find a thermal means for shortening the beam back to its original shape. Specify what temperature change is needed for the desired correction.
2) Please find a mechanical means to achieve the same objective. Specify the magnitude of the force needed.

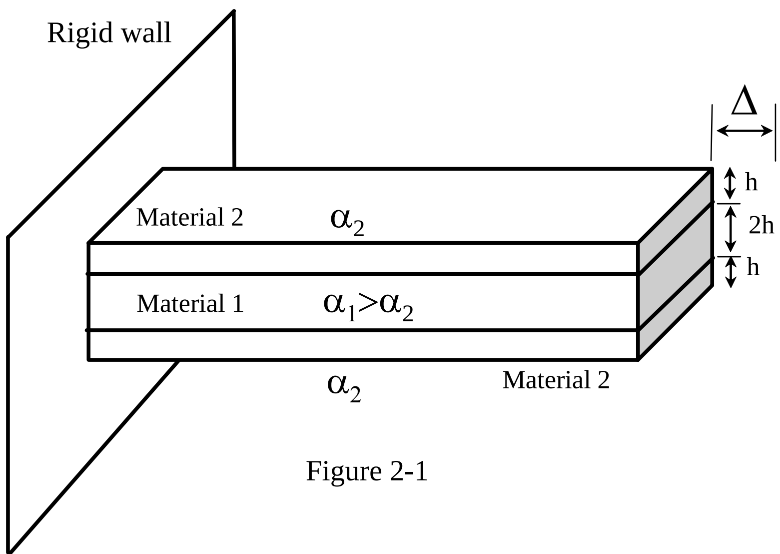

Figure 2-1

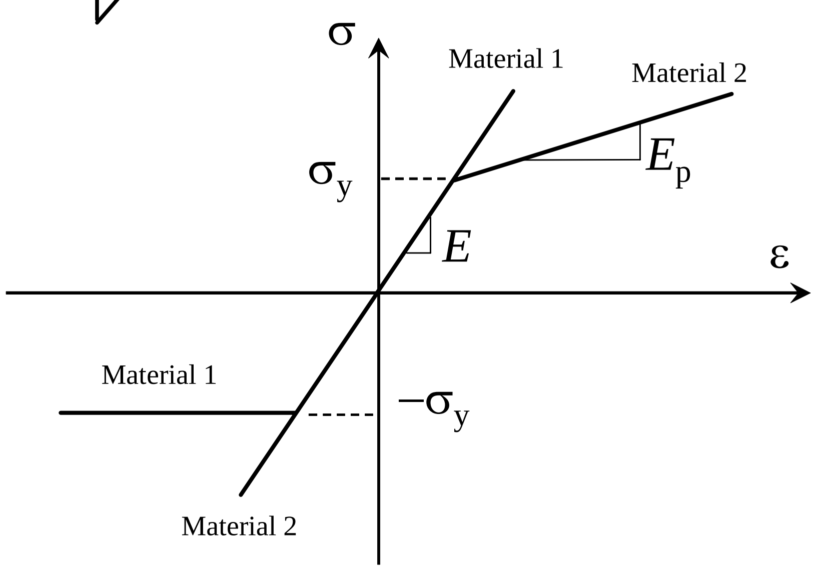

Figure 2-2

Problem 3

Consider a specimen in four-point bending as shown in Figure 3-1.

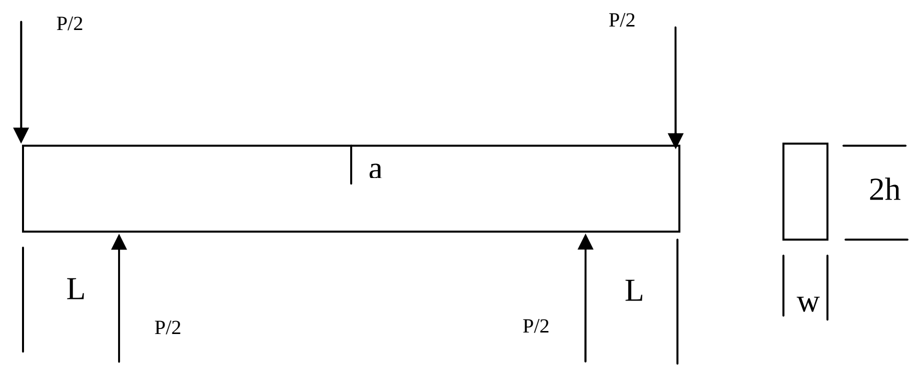

Figure 3-1

Given:

L=20mm
w=3mm
h=3mm
σ_{ys}=100MPa
E=100GPa
K_{Ic}=5MPa $m^{1/2}$
a=0.3mm

1) If the material does not yield, at what load P will the crack begin to propagate? Please explain the conditions of small scale yield. (Calculate the size of the plastic zone as part of your explanation. If you have memorized an equation for the plastic zone size, explain where it came from.)

2) If there were no crack, at what load would the beam yield?

3) If the cracked beam is loaded until it fails, will it fail by yielding or will it fracture first (assume small scale yield conditions)?