GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Semester 2000

Mechanics of Materials
EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

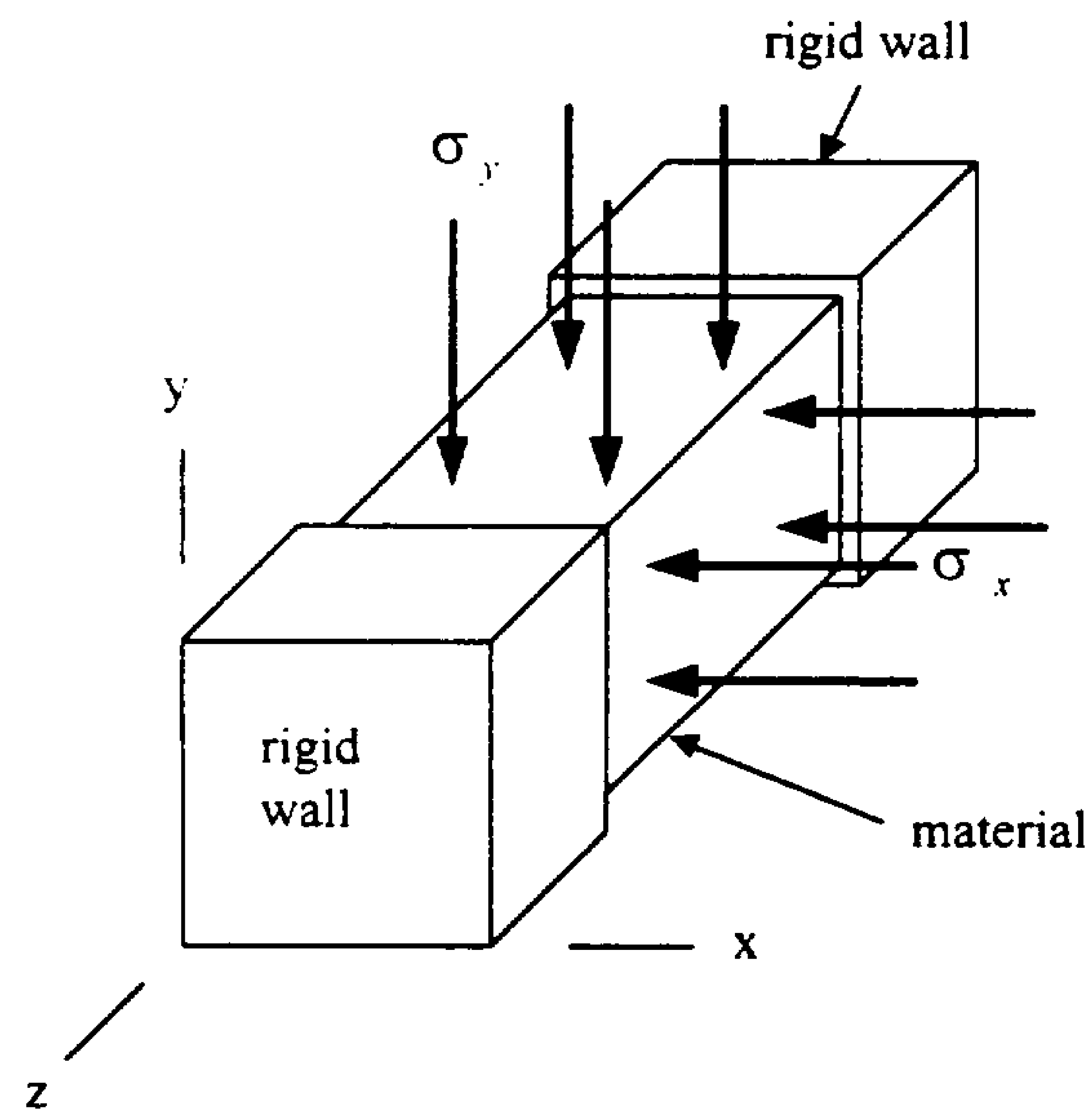
Problem I

A block of material is stressed in the *x*- and *y*-directions as shown, but rigid walls prevent deformation in the *z*-direction. The ratio of the two applied stresses is a constant, so that $\sigma_y = \lambda\sigma_x$. Answer the following by deriving equations expressed in terms of σ_x, λ, and the elastic constants of the material:

a) Does a stress develop in the z-direction? If so, how is it related to σ_x and the other constants involved?

b) Determine the stiffness $E' = \sigma_x / \varepsilon_x$ for the *x*-direction.

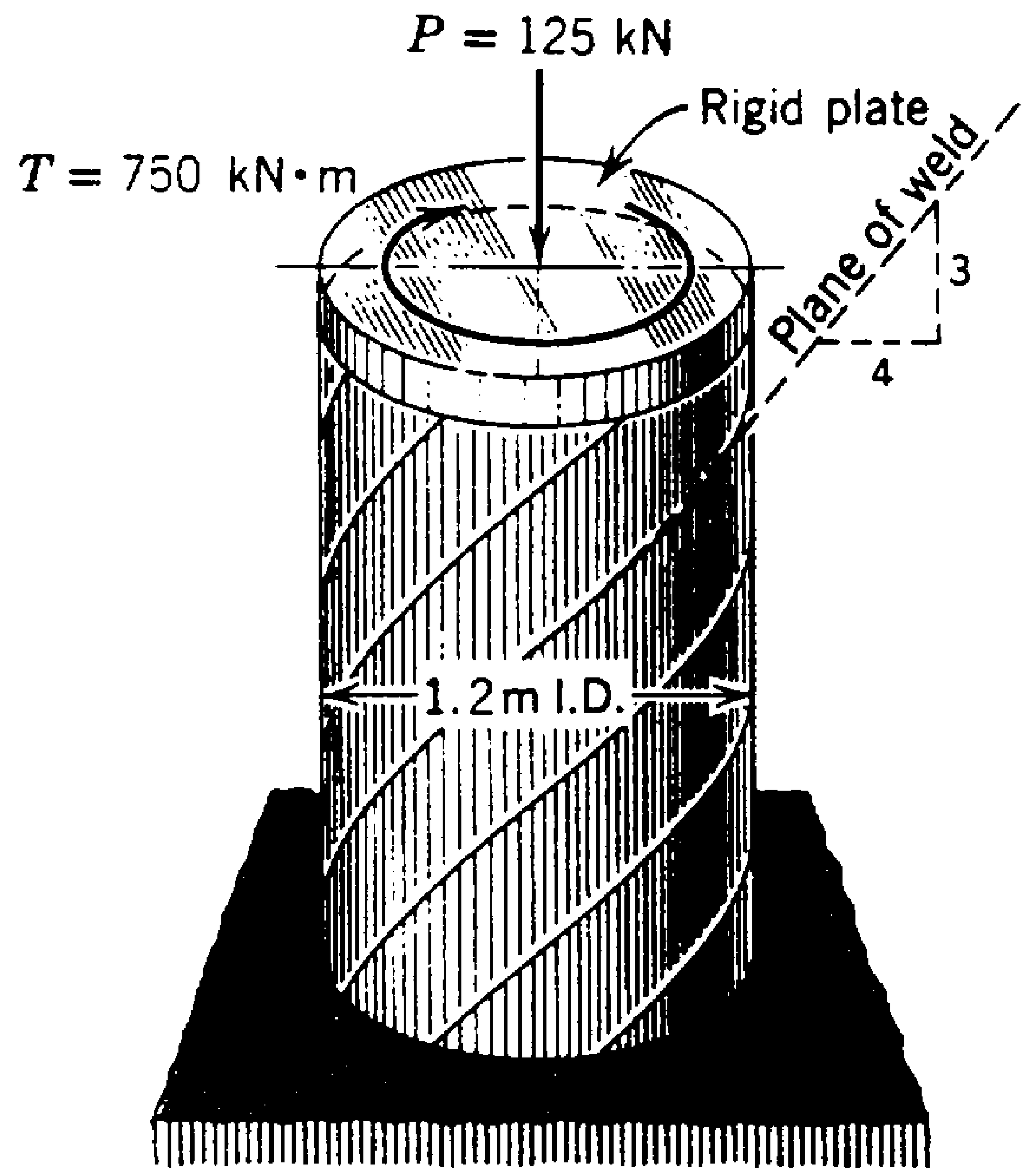
c) Compare this apparent modulus E' with the elastic modulus E from a uniaxial test. (Suggestion: consider λ values of -1, 0, and 1 and assume $\nu = 0.3$.)



Problem II

A thin-walled cylindrical pressure tank is fabricated by butt-welding 15-mm thick plate with a spiral seam as shown in the figure. The pressure in the tank is 2.5 GPa. Additional loads are applied to the tank through a rigid end plate as shown in the figure. Please determine

(a) The normal and shear stresses on the plane of the weld.
(b) The principal stresses and the maximum shearing stress at a point on the inside surface of the tank.
(c) The continuity conditions across the weld line.



Problem III

Figure below shows a symmetric structure made of two materials. The properties of the materials are given in the table below:

	Material	Young's Modulus (GPa)	Coefficient of Thermal Expansion (CTE)	Thickness (mm)	Length (mm)
Material 1	Silicon	128.9	2.89×10^{-6} /°C	0.5 (each side)	40
Material 2	Pyroceram Glass	94	0.3×10^{-6} /°C	5.0	40

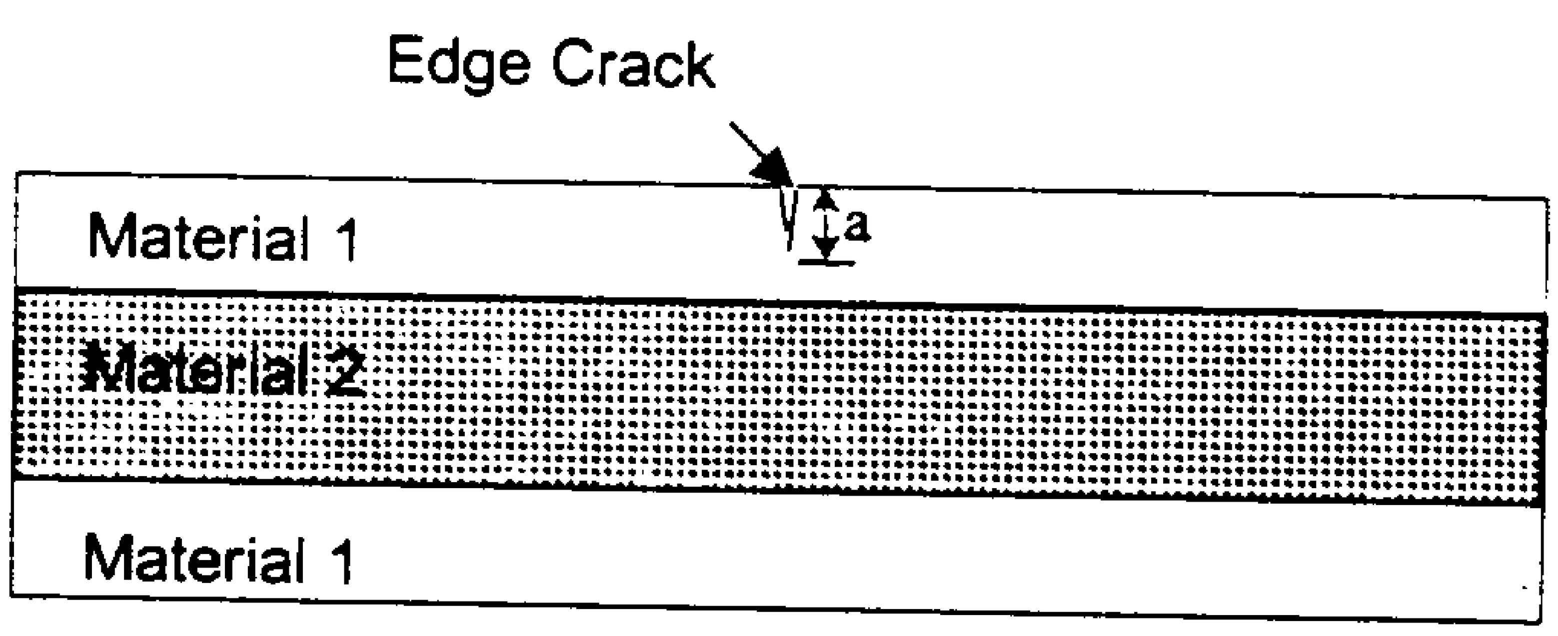
Material 1 is assembled on Material 2 at a stress-free temperature of 320 °C, and the structure is then cooled to room temperature of about 20 °C.

The assembly process and handling could create edge defects or flaws as shown in the figure. The fracture toughness of silicon K_{IC} is about 25.9 $N/mm^{3/2}$ and can be related to the edge crack as:

$$K_{IC} = 1.12\sigma\sqrt{\pi a}$$

where a is the flaw size as shown in the figure.

- State all your assumptions
- Determine if Material 1 is likely to crack, upon cooling to room temperature. If so, determine the maximum flaw size that the structure can withstand without cracking
- Following the assembly process, the structure is thermal cycled between –40 to 125 °C to evaluate its integrity. State whether the chances of cracking are more at –40 or at 125 °C. You need not work through the numbers. Provide a qualitative answer.



Problem IV

An external torque T_A is applied to the composite circular shaft (r_i and r_o inside and outside radii) shown above. The shaft has two longitudinal sections: (a) a top annular section of a single material (shear modulus G_1); and (b) a bottom composite section consisting of two materials with different moduli (annular material G_1, core material G_2). There is no slippage between the two materials and the bottom end of the shaft is welded to the support.

1. Derive an expression for the **twist angle** (ϕ_A) for the top end of the shaft in terms of the applied torque, shear moduli, and shaft dimensions. Clearly state your assumptions.
   (Note: You may express your answer in terms of the polar moments of inertia)

2. Consider two small elements on the **outer surface** of the shaft (away from the top and bottom ends), one on each of the two longitudinal sections a and b. Find the **ratio of radii** such that the shear stress on the element on section a (τ_a) is twice the shear stress on the element on section b (τ_b).