

JUL 16 1997

**RESERVE DESK**

**GEORGIA INSTITUTE OF  
TECHNOLOGY**

The George W. Woodruff  
School of Mechanical Engineering

**Ph.D. Qualifiers Exam - Spring Quarter 1997**

Mechanics & Materials

EXAM AREA

**Assigned Number (DO NOT SIGN YOUR NAME)**

- Please sign your name on the back of this page—

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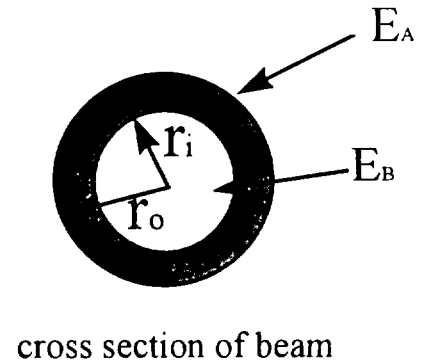
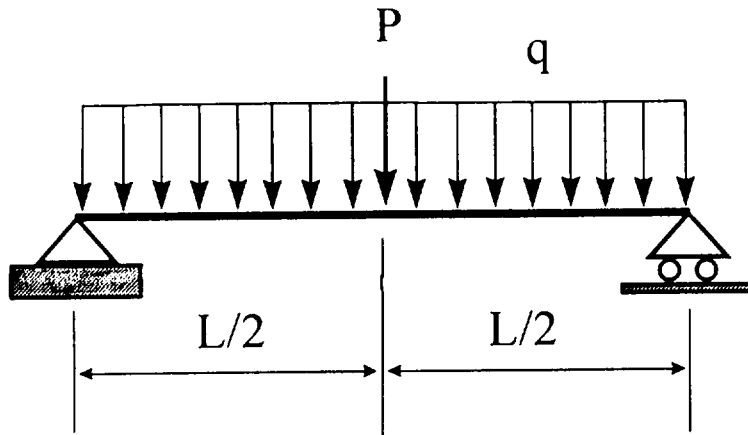
Please **print** your name here.

The Exam Committee will get a copy of this exam and will not be notified whose paper it is until it is graded.

**Mechanics of Materials**

**Qualify Exam (written)**

A simply supported beam is subject to a uniformly distributed load with intensity  $q$  and a concentrated load  $P$ . The beam has a composite cross section as shown. Assume the materials are perfectly bonded at their interface. (1) Write down the differential equation for the flexure curve of this beam; (2) Find the total amount of strain energy stored in the beam, (3) Find the maximum deflection with or without solving the differential equation.



**Z-2.** Please draw a free body diagram showing your moment balance. Use this to derive equations for the torque induced stresses in the thin walled tube.

**Z-3.** Please draw Mohr's circle. Use the geometry to derive equations for the principal stresses and the maximum shear stress. Discuss how the principal stresses and directions vary with time.

**Z-4.** What is the maximum shear stress? (Discuss in terms of the Mohr's circle and time.)  
How is this related to yield?

### Mechanics of Materials Qualifier Problem

2

The company you work for has just manufactured two rather expensive pressure vessels connected by a pipe. When this system is put in service, a valve will be periodically opened and closed, causing the pressure to fluctuate as

$$P = P_0 \cos \omega t$$

The pressure vessels are very heavy and stiff, so the tube is constrained to zero strain in the length direction. When the system was assembled, the two vessels were not properly aligned. A few quick measurements show that aligning the pressure vessels places a torque  $T$ , on the tube. This was not considered in the original design calculations. You, as the engineer, are asked if this system will be safe or if it must be scrapped.

The first thing you are to look at is the safety factor against yield. Your boss has asked you to derive a formula for the principal stresses and principal directions as a function of time. The tube can be modeled as a thin walled tube with internal pressure and with a torque about its axis.

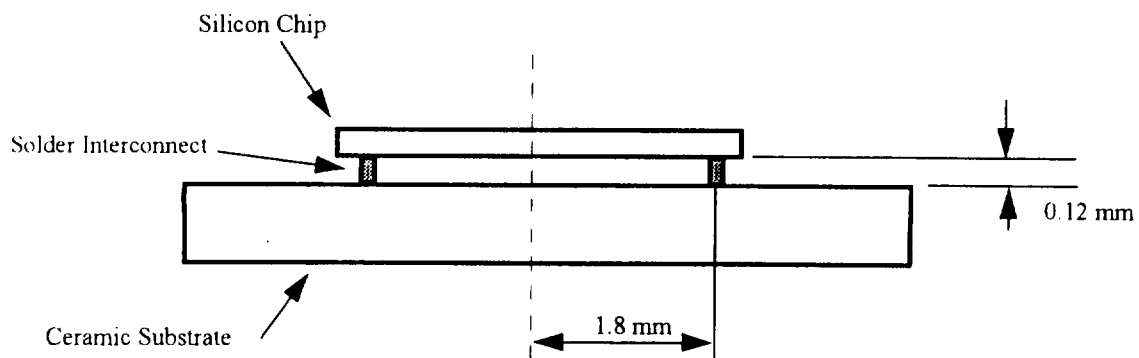
1. Please draw a free body diagram showing your force balance for the pressure. Use this to derive equations for the pressure induced stresses in the thin walled tube. (Note that the end constraints are not the typical textbook example.)

3

The figure below shows a silicon microelectronic chip mounted on a ceramic substrate. The electrical contact between the chip and the ceramic substrate is established through solder interconnects as shown in the figure.

The electronic packaging assembly is used in an automotive ignition module where the temperature could go up to 120 °C when the automobile is driven. The silicon chip has a coefficient of thermal expansion  $2.8 \times 10^{-6} / ^\circ\text{C}$  and the ceramic substrate has a coefficient of thermal expansion  $6.5 \times 10^{-6} / ^\circ\text{C}$ .

Assume that the entire assembly is stress-free at an ambient reference temperature of 20 °C. Also, assume that the assembly does not warp and that there are no temperature gradients or transients.



- 1) Sketch the deformed assembly and determine the shear strain in the solder interconnect when the automobile is driven.
- 2) According to select theories, the number of fatigue cycles can be related to the strain range using the following power equation.

$$N_f = \frac{1.2928}{4} (\Delta\gamma)^{-1.96}$$

where  $N_f$  is the number of fatigue cycles and  $\Delta\gamma$  is the shear strain range. Assume that the automobile is driven twice a day. Determine how many days the automobile will run before the ignition module needs to be replaced.

- 3) If you were asked to increase the fatigue life changing only the dimensions, what parameters would you change?

State all your assumptions clearly.

4

A thin, linear elastic circular membrane is stretched uniformly and then clamped at its outer edge as shown. The radius of the membrane is  $a$ . A rigid circular insert of radius  $b < a$  is attached to the membrane as shown and used to apply a torque  $T$ .

- State clearly the usual assumption(s) made in analyzing the torsion of linear elastic uniform circular cylindrical shafts subjected to a constant torque  $T$ .
- Assume that these assumptions hold for the problem described. Calculate the relationship between the torque  $T$  and the rotation of the insert  $\theta$ .

