GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

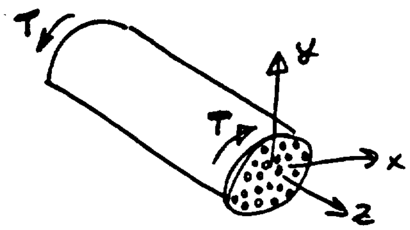
Ph.D. Qualifiers Exam - Spring Quarter 1995

MECHANICS OF MATERIALS
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

-- Please sign your name on the back of this page --

1. (a) Consider combined bending and torsion of a fiber-reinforced composite circular shaft, with fibers running along the axis of the shaft. The anisotropic linear elastic stress-strain relation is given below. Which are the nonzero stress components in cylindrical (r,θ,z) coordinates? How would one go about computing the maximum shear stress in the shaft under an applied torque, T, and where would this occur?



$$\begin{bmatrix}\epsilon_x\\ \epsilon_y\\ \epsilon_z\\ \epsilon_{yz}\\ \epsilon_{xz}\\ \epsilon_{xy}\end{bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu'}{E'} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu'}{E'} & 0 & 0 & 0 \\ -\frac{\nu'}{E'} & -\frac{\nu'}{E'} & \frac{1}{E'} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2G'} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2G'} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1+\nu}{E} \end{bmatrix} \begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{bmatrix}$$

(b) Can the shear stress vary nonlinearly with radius in this case? Why or why not?

1 (cont.)

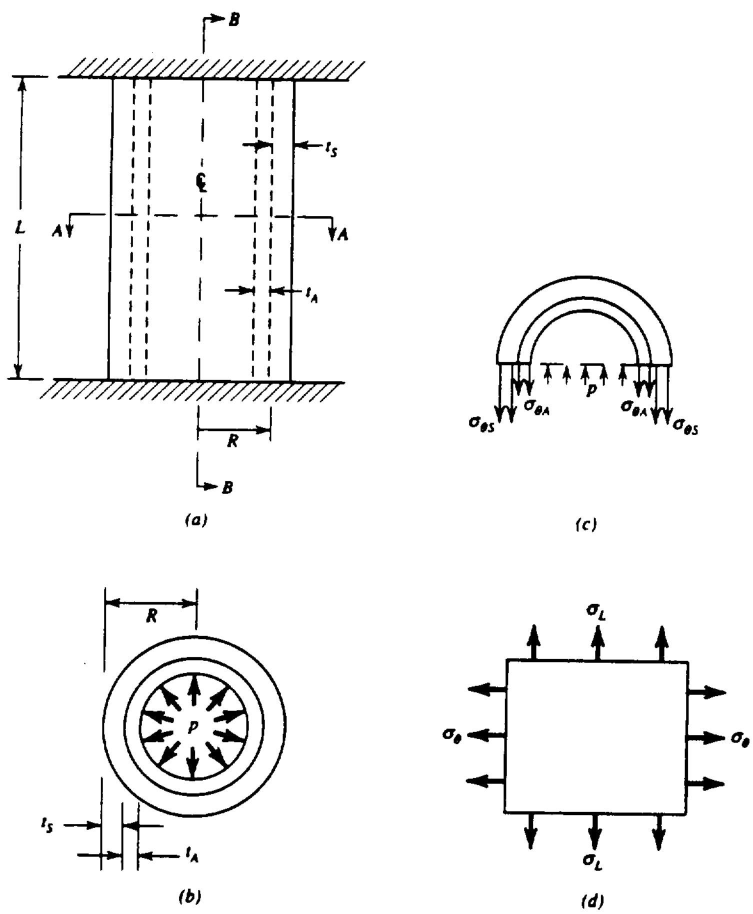
(c) If the shaft is subjected to combined bending and torque, is it permissible to superimpose the solutions for the stresses at each point due to the applied bending moment, **M**, and applied torque, **T**, with each solution determined independently? Why or why not? How would each of these solutions be determined?

(d) Please outline how each of the five independent elastic constants E, E', ν, ν' and G' might be determined using simple mechanical tests and strain measurements.

2 Consider a composite cylinder of length L formed from an inner cylinder of aluminum with outer radius R and thickness t_A, and an outer cylinder of steel with inner radius R and thickness t_S; $t_A \ll R$, $t_S \ll R$. The composite cylinder is supported snugly in an upright, unstressed state between rigid supports. An inner pressure p is applied to the cylinder and the entire assembly is subjected to a uniform temperature change ΔT. Determine the stresses in both the aluminum and the steel cylinders for the case

$$t_A = t_S = t = 0.02\,R.$$

For aluminum, $E_A = 69$ GPa, $\nu_A = 0.333$ and $\alpha_A = 21.6 \times 10^{-6}$ per °C. For steel, $E_S = 207$ GPa, $\nu_S = 0.280$ and $\alpha_S = 10.8 \times 10^{-6}$ per °C. Subscripts A and S refer to the aluminum and steel, respectively.



(a) Composite cylinder. (b) Cross section A-A. (c) Longitudinal section B-B. (d) Cylinder element.

3. The stress intensity factor for a thumbnail crack in a plate subjected to tension can be calculated from

$$K_I = \frac{1.12}{\sqrt{Q}}\sigma\sqrt{\pi a}$$

where Q is termed the shape factor which depends on a and c. The figure below graphically shows this dependence for various ratios of nominal applied stress, σ, to the yield strength, σ_y.

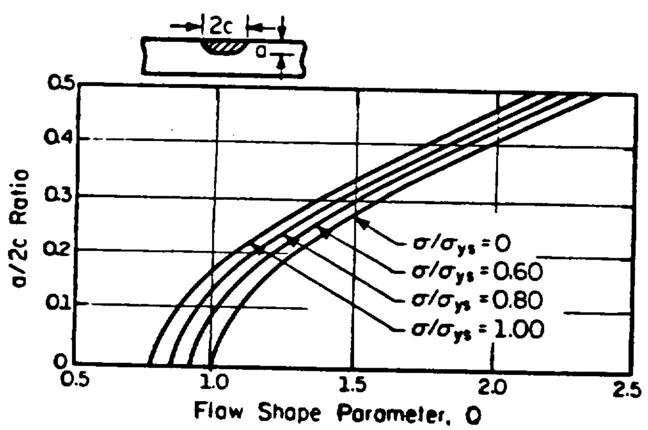
Now, consider a pressure vessel made from a material that has a fracture toughness of $K_{Ic} = 60\text{ksi}\sqrt{\text{in}}$, and a yield strength of $\sigma_y = 85\text{ksi}$ at the operating temperature. Radius of the vessel is $r = 18\text{in}$, the wall thickness is $t = 0.75\text{in}$, and the operating pressure is $p = 2\text{ksi}$.

It is required that the vessel "leak-before-burst." In other words, the crack must be able to grow through the wall thickness before fast fracture occurs. This allows the gas or liquid in the pressure vessel to escape and be detected before an unstable condition develops.

It is assumed that the pressure vessel will be inspected periodically with an ultrasonic technique that can detect a crack with a surface length $2c < 0.5\text{in}$. Please answer the following questions.

(a) Will the pressure vessel leak before burst when the surface length of the crack is smaller than 0.5in?

(b) What is the largest value of the surface crack that can develop and still maintain the leak-before-burst criteria?



4. Shown below is a circular thin membrane of uniform thickness t and radius a. The membrane is stretched and clamped at r=a. The rigid insert of radius b is used to apply a torque T to the membrane as indicated. Assume that the membrane material has shear modulus G.

a) Write down the assumption(s) made for the usual linear elastic torsion of uniform circular shafts subject to constant torque.

b) Assuming these assumptions still hold, determine the relationship between torque T and twist ϕ.

c) Does your analysis still work if the membrane is not stretched first? Why or why not? Discuss.