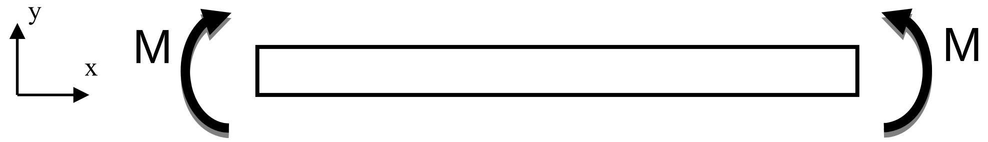
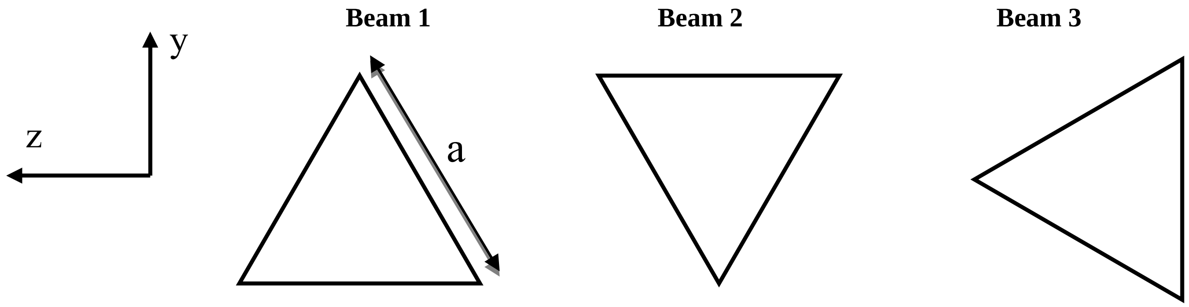
NAME: ______________________________

1. A prismatic beam is under pure bending as shown below. The beam's cross-sectional shape is an equilateral triangle. Consider three possible beam configurations all bending about the z-axis as illustrated. Show work to justify your answers.





(a) If the material and size of the triangular cross section are the same for all three beams, please determine which beam(s) is (are) the most stiff and the most compliant for the bending moment shown.

(b) Which is (are) the strongest beam(s) and which is (are) the weakest? Assume the material strengths in tension and compression are the same.

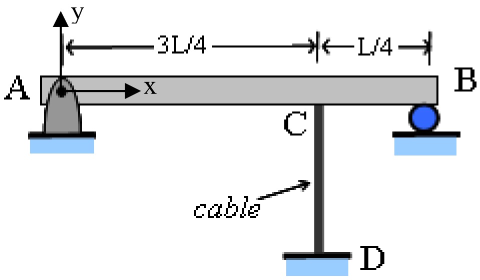
(c) If the material strength in compression is 10 times larger than that in tension, does your answer for part (b) change? If so, how does your answer change?

(d) If Beam 1 is made of a ductile material and is sufficiently loaded so that yielding occurs at some points in the beam and then is unloaded, what will be the resulting shape of the beam? For this part, words and drawings are sufficient. Also, assume strengths in tension and compression are the same.

2. During your graduate studies at Georgia Tech you design a new material and you think this material has a broad range of aerospace applications. Given your Yellow Jacket pride, you call this material Buzyanium. You are able to sell the idea, and in 2015 the surface of the wing of the ‘next generation’ spacecraft that NASA will launch will be made of Buzyanium. Based on the experiments you performed in your lab, you determined that the material is elastically isotropic and that E = 10 GPa and ν = 0.4. The strains at a critical location on the wing surface are monitored at a given point by means of three strain gauges arranged at angles relative to one another according to α_1=0°, α_2=60°, α_3=120°. During a certain maneuver, the following strains were recorded: ε_{0^o} =200 X 10^{-6}, ε_{60^o} = 100 X 10^{-6}, and ε_{120^o} =300 X 10^{-6}.

   (a) Determine the strains relative to the *xyz* coordinate system if the *x*-axis is oriented at 0° and the y axis is at 90° in the plane of the strain gauges.
   (b) Determine the principal strains, directions of the principal strains, the maximum shear strain, and the orientation of the plane of the maximum shear strain.
   (c) Find the components of the stress tensor in the coordinate system corresponding to the principal stress frame (*x’y’z’*), and clearly show in a figure how this frame is rotated relative to the xyz frame that defines the orientation of the strain gauges. Find the components of the stress tensor in the *xyz* frame.

3. A simply-supported beam AB of length L is connected to a cable at point C as shown in the figure below. The second moment of area of the beam is I, the cross sectional area of the cable is A and the length of the cable is L_{CD}. The cable is initially taut but without any residual force.



(a) If the temperature drops by ΔT, what is the force, F_c, in the cable with respect to the above mentioned parameters? Both the cable and the beam are made of the same material. Assume that under a temperature change *without* the cable attached, the bottom surface of the beam would stay horizontal and fixed in terms of its vertical (y) position.

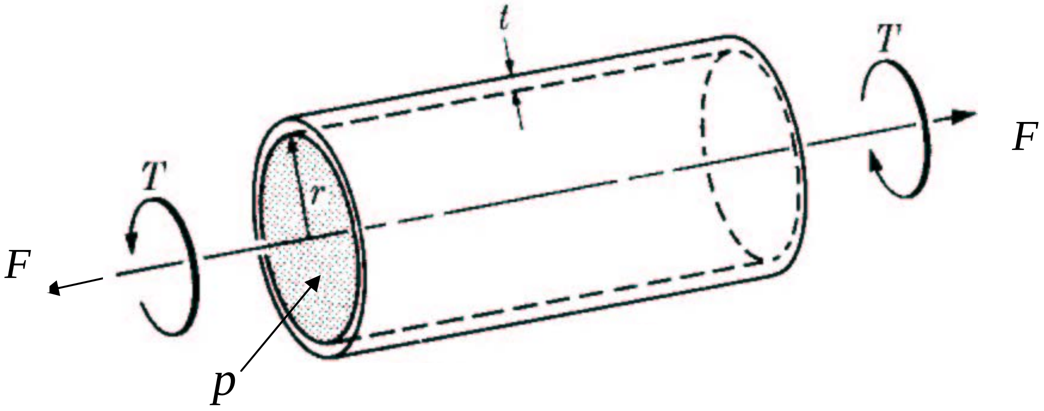
(b) List your assumptions.

(c) Recommend changes that can be made to the material properties so that the overall deflection at C is minimized.

4. A steel tube (assume isotropic linear elasticity with E = 200 GPa, ν = 0.3) has an internal diameter of 500 mm and a wall thickness of 30 mm. The yield strength of this steel is 500 MPa. It is subjected to multiaxial loading consisting of:

- an internal pressure *p*
- a torque *T*
- an axial force F

For purposes of this analysis, assume that internal pressure does not contribute to axial stress in the tube.



Within the section of interest, parts (a)-(e) below pertain to the given geometry & loading conditions:

(a). Determine the factor of safety on yielding for the tube based on both the von Mises and Tresca yield criteria. Please compare these values. What are the limitations on these two criteria?

(b). If no torque is applied, is there some relation between *F* and *p* such that yielding would ***never*** occur according to the Tresca criterion, irrespective of the magnitudes of *F* and *p*? Explain.

(c). If the applied loading (*T*, *F*, *p*) increases with only *F* applied up to the point of initial yielding, i.e., (*0*, *F*, *0*) up to this point, determine the value of *F* at initial yielding.

(d). Considering the loading with pure *F* in part (c) carried on ***past*** the point of initial yield, if torque *T* and internal pressure *p* are subsequently applied along with continued increase of *F* to a point (T_e, F_e, p_e), would the material be expected to exhibit the same value of effective stress at this terminal point as would be realized by loading up to this point proportionally from the outset (i.e., even during the elastic region), i.e., $p = \lambda_1 F = \lambda_2 T$ where λ_1, λ_2 are constants? Explain your answer from a physical viewpoint and from the perspective of incremental plasticity theory.

(e). Suppose there is no internal pressure. Would the driving force for growth of a through-thickness longitudinal crack in this pressure vessel be sensitive, to first order, to the applied axial force *F*? Torque *T*? Please explain, including clear discussion regarding modes of crack extension in LEFM.