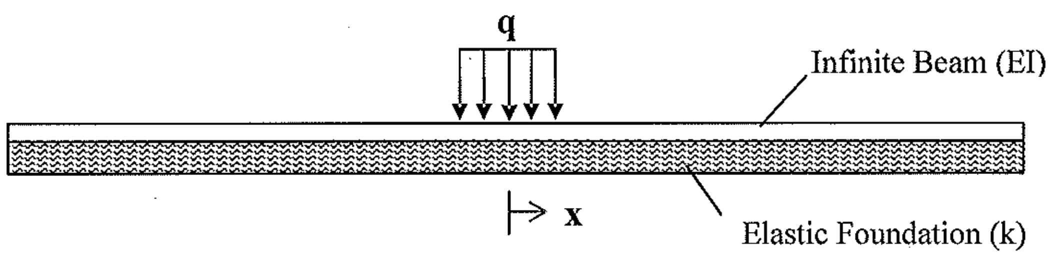
1. The infinite beam shown below is permanently attached to an elastic foundation. A practical example of this problem is a railroad rail (beam) supported by nonrigid ties and ballast below. The beam supports can be idealized as a continuous elastic foundation. The foundation has stiffness **k** that is finite and the beam is straight with elastic properties **EI**. The distributed load **q** is static. Let w(x) represent the beam deflection.
    A. Draw a free body diagram of the beam near x = 0.
    B. Show the forces and moments that act on a differential element of the beam near x = 0.
    C. Write equilibrium equations to find expressions for dV/dx and dM/dx.
    D. Write a fourth order governing equation for the beam deflection w(x). Do not solve.
    E. Replace the distributed load with a concentrated load **P** at x = 0. Draw estimated plots of beam deflection, bending moment, and transverse shear force for the length of the beam shown. Do not solve.



2. In a supersonic wind tunnel, long, thin-walled, cylinders (radius $R = 0.9$m, thickness t) are submitted to cycles of internal pressure. The maximum pressure during normal operating conditions is $P_{max} = 5000$ kPa. The goal of this problem is to determine the optimal thickness t of the cylinders such that no failure can occur for an internal pressure as high as twice the P_{max} value. Two different failure events are studied:
   - plastic rupture by yielding
   - fracture by rapid crack propagation

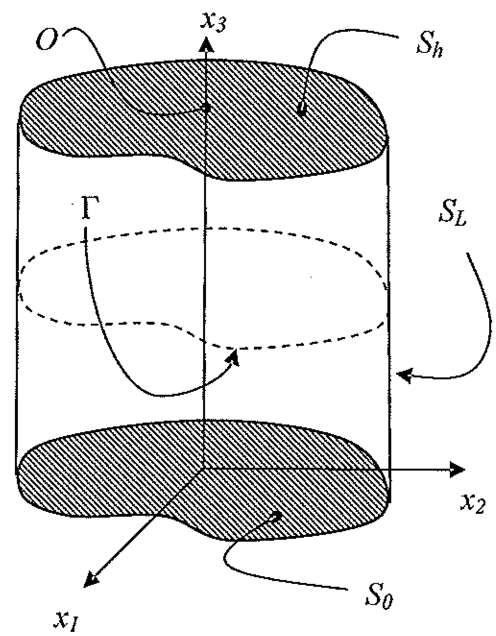
   1. Determine the stress tensor for the cylinder submitted to an internal pressure P.
   2. Plastic rupture by yielding: Assuming the cylinder is made of an elastic-perfectly plastic material (yield strength, σ_y), determine the minimum thickness necessary to prevent this failure event.
   3. Fracture by rapid crack propagation: Assume that the most severe initial defects present within the cylinders are embedded circular cracks of radii a, for which the stress intensity factor is approximately: $K_I = \sigma_{\theta\theta}(\pi a)^{1/2}$. The fracture toughness of the cylinder material is K_{IC}. Determine the minimum thickness necessary to prevent this failure event.
   4. Determine the critical radius, a_c, for which both failure events occur simultaneously.
   5. From a safety viewpoint, which failure event is worse? Answer this question by qualitatively explaining what happens when the internal pressure is gradually increased for a cylinder with an initial defect of radius a_0, for the following cases:
      a. $a_0 < a_c$
      b. $a_0 > a_c$
   6. What is the maximum thickness t that shouldn't be exceeded to prevent the worse failure event? Use a safety factor of 2.
   7. The following two materials are considered for the cylinder:
      a. Steel: $\sigma_y = 1000$ MPa, $K_{IC} = 170$ MPa.m$^{1/2}$
      b. Aluminum alloy: $\sigma_y = 400$ MPa, $K_{IC} = 25$ MPa.m$^{1/2}$

      Which material should be selected? What would be the optimal thickness for that material?
   8. Are there any additional failure events that should be considered to predict the structural reliability of the cylinders?

3. You have a unidirectional composite material. Assuming that the fiber modulus is 50 msi and the strength is 500 ksi, the matrix modulus is 0.5 msi and the strength is 20 ksi. The fiber volume fraction is 0.6. (msi = million pounds per square inch; ksi = thousand pounds per square inch)

a) Derive the Rule-of-Mixtures for the longitudinal modulus.
b) Using the fiber and matrix properties, calculate the composite stiffness in the fiber direction.
c) Calculate the ultimate strength of the composite in the fiber direction.

4. Given a cylindrical solid whose axis is parallel to x_3 and is limited by the lateral surface S_L and the top and bottom by surfaces S_h and S_0, respectively (see the figure at right). The contour shown, Γ, is the intersection of the lateral surface of the solid, S_L, and a plane normal to x_3. It is known that all components of the stress tensor except σ_{13} and σ_{23} are null at all points and that all body forces are negligible.



(a) Show the a function, $\phi = \phi(x_1, x_2)$, exists such that:

$$\sigma_{13} = \frac{\partial \phi}{\partial x_2} \quad \text{and} \quad \sigma_{23} = -\frac{\partial \phi}{\partial x_1}$$

(b) Given that there is no loading on the lateral surface, S_L, show that the function $\phi(x_1, x_2)$ is constant everywhere on the contour Γ.

(c) Calculate the equivalent force and moment vectors, $\vec{F}$ and $\vec{M}$, at the point O due to all surface tractions exerted on the top surface S_h.

(d) If we choose the contour, Γ, to be an ellipse described by, $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 1$, and we define the function $\phi(x_1, x_2)$ to be $\phi = \lambda\left(\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2}\right)$, repeat part (c).