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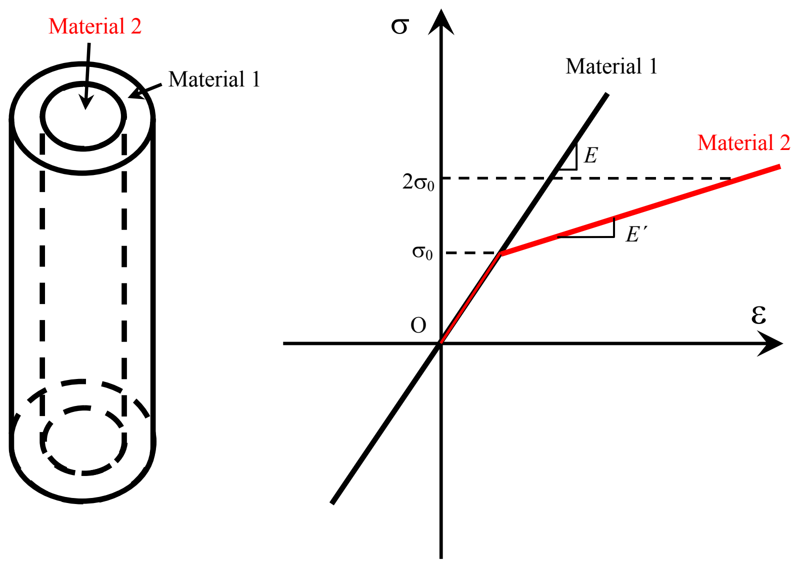
Mechanics of Materials
PhD Qualifying Examination
Fall 2005

Please work all four problems – 25 points each (show ALL work)

1. (a) Determine the required wall thickness of a thin-walled cylindrical pressure vessel with diameter 2.0 m that must contain a maximum pressure of 3000 kPa. It is to be fabricated from a material with a yield strength of 600 MPa. Use a factor of safety of 2.0. Show all work and provide any assumptions made.

(b) Safe pressure vessels are designed to either "yield before break" (e.g., small pressure vessels that will bulge when overloaded) or "leak before break" (e.g., larger pressure vessels that will leak at a site of a crack before catastrophic fracture). Is the previous calculation sufficient to ensure that there will not be a brittle fracture? If not, please outline the analysis you would perform providing all necessary equations and any other recommendations. First consider the "yield before break" case, then consider the "leak before break" case.

2. A composite wire is being designed for use as interconnects in devices. This wire has an inner core and an outer cladding of equal cross-sectional areas made of two different materials as shown in the figure below. The application calls for perfect bonding at the core-cladding interface. The wire is expected to be subjected to only tensile external loading. Material 1 is always linear elastic. Material 2 is elastic-plastic and part of its stress-strain curve is shown in the figure. Question (1) below concerns the behavior of material 2 when it is deformed alone. Questions (2)-(4) concern the behavior of the composite wire. Clearly state your assumptions or approximations in carrying out the following analyses.



(1) If material 2 is stretched to a stress of $2\sigma_0$, complete the stress-strain trajectory for the process of unloading from that point and subsequent loading in compression to a stress of $-2\sigma_0$ if material 2 shows:
  (a) pure isotropic hardening
  (b) pure kinematic hardening

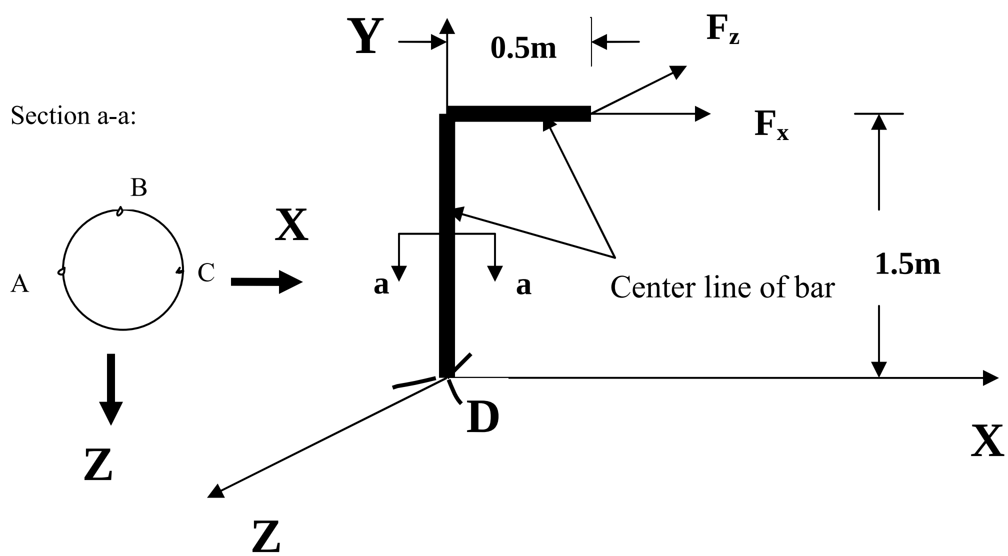
(2) For each of the two types of hardening of material 2 above, describe (analytically or graphically with key parameters specified) the stress-strain behavior of the wire over one cycle comprised of tensile loading to $2\sigma_0$, followed by unloading to zero stress.

(3) What is the free length of the wire after unloading compared with the initial length in the above scenarios?

(4) What are the residual stresses in the core and the cladding after unloading in the above scenarios?

3. A 2m circular steel bar (E=200x10^9 N/m^2 , ν=0.25) of radius 5 cm is bent at 90^o in the (x,y) plane as shown in the sketch below which shows the *centerline* (thick line) of the bar. The bar is anchored at D and loaded at the end by forces F_x and F_z as shown in the sketch. Section a-a contains points A, B and C as indicated in the sketch, also shown below.

a) Determine the stresses at points A, B and C.
b) Show the stresses at points A, B, and C as they act on a properly oriented element. Be sure and *clearly* indicate the orientation of elements you sketch.
c) If you were to consider plastic failure of the bar at the section containing points A, B and C, which point would be the most critical? For failure, assume that the maximum shear stress criterion holds.

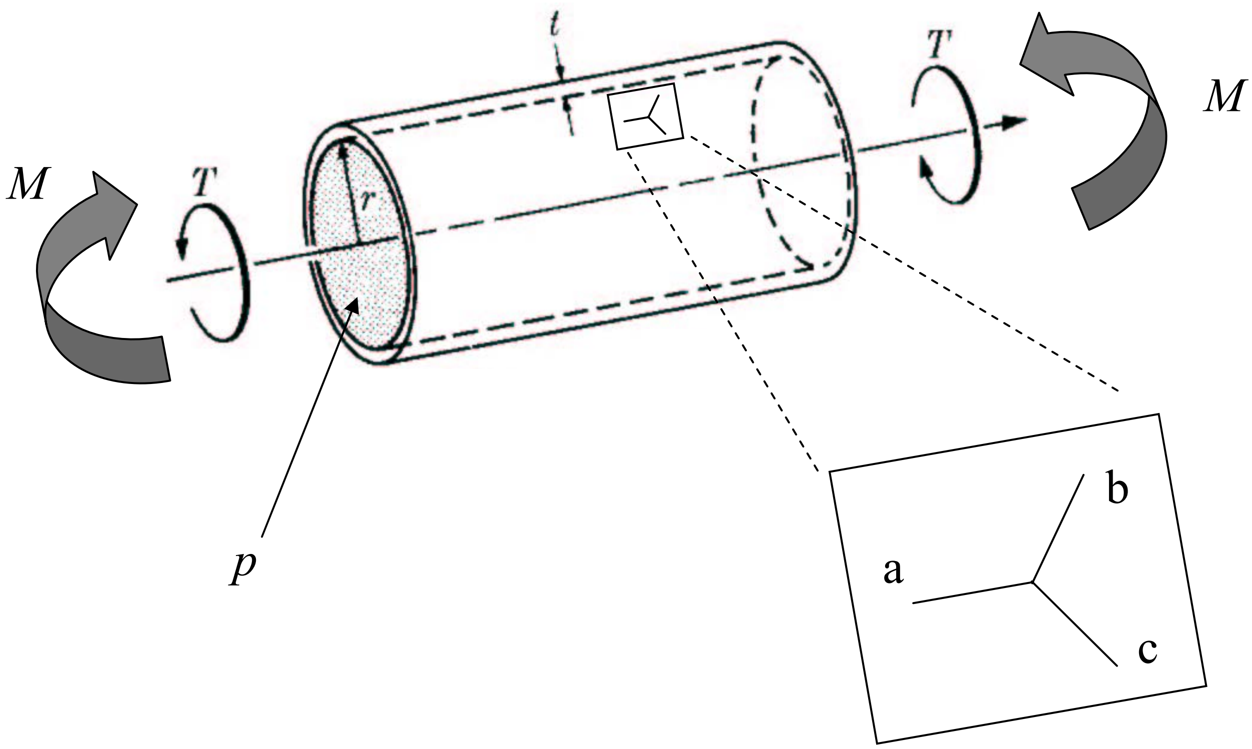


4. A brass tube (assume isotropic linear elasticity with E=100 GPa, ν=0.35, α=20x10^{-6}/K) has an internal diameter of 460 mm and a wall thickness of 25 mm. It is subjected to complex loading consisting of:

- internal pressure p of 2 MPa
- torque T of 40 kN-m
- bending moment M of 25 kN-m

For the purpose of this analysis, assume that the tube is capped at both ends far from the section of interest.

Within the section of interest:



a) Determine the <u>location</u> and <u>value</u> of the *maximum principal stress* σ_{max}

b) Determine the <u>location</u> and <u>value</u> of the *minimum principal stress* σ_{min}

c) Determine the magnitude and orientation of the *maximum shear strain* γ_{max} on the *outside* of the tube

d) Assume that the 60° rosette shown is on the very "top" of the tube. Find the strain measured by each of the three rosettes.

e) If the temperature were now increased by 50K, what would be the new value of γ_{max} on the outside of the tube?