GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff

School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Semester 2002

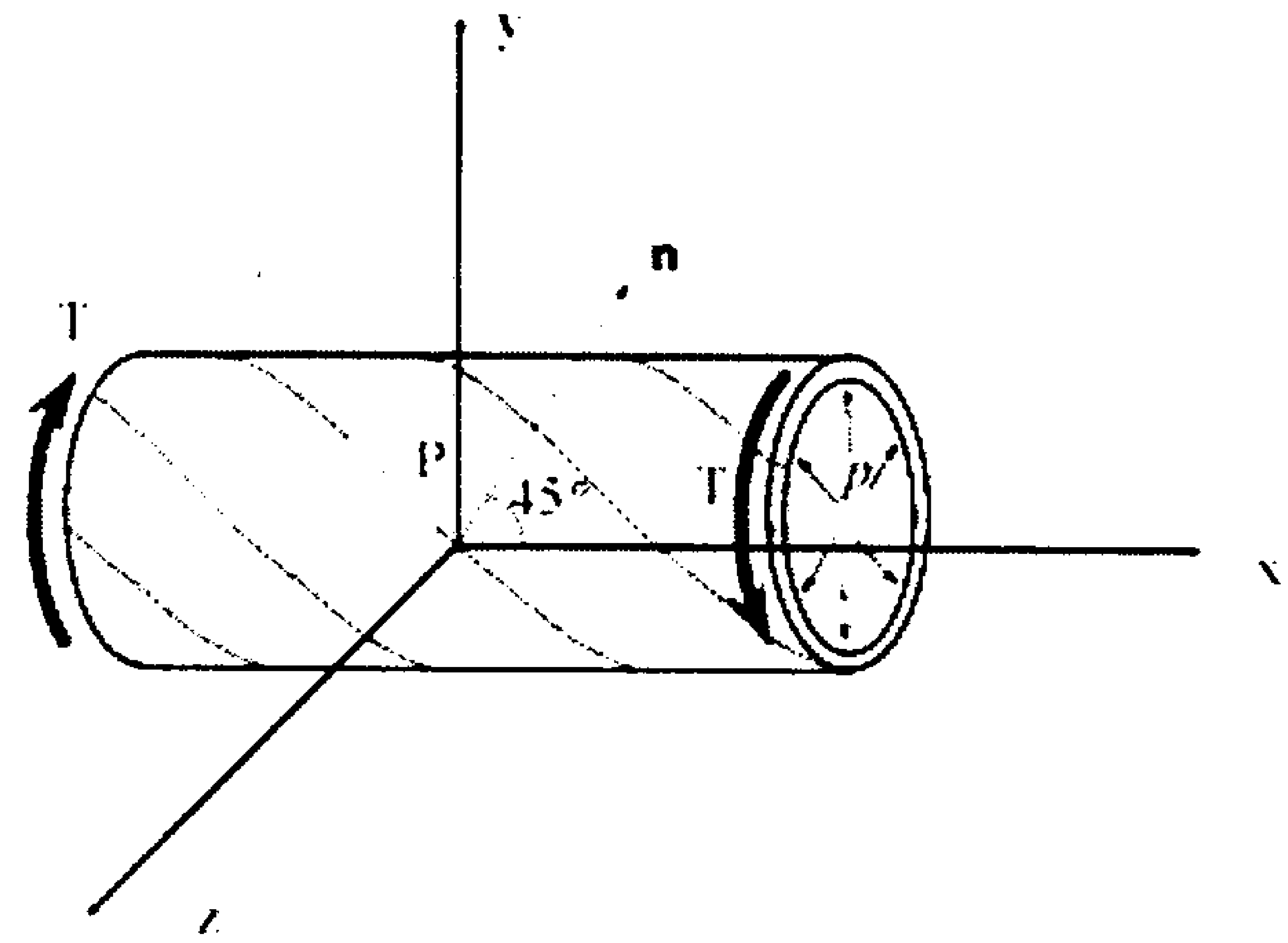
Mechanics of Materials

EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

1. A cylindrical, close-ended, thin-walled pressure vessel is subjected to loading with internal pressure p and torque T. The vessel has an average radius of 0.2 m. This vessel is manufactured by welding a sheet strip along a helical weld line which is inclined at an angle of 45° with respect to the x-axis. The elastic constants for the vessel are $E = 70$ GPa and $\nu = 0.3$. It has a yield stress of $\sigma_0 = 500$ MPa. Furthermore, the weld is known to fail under the condition $\frac{1}{2}(1+\operatorname{sgn}\sigma_n)\sigma_n^2 + 4\tau_n^2 \geq (300\text{ MPa})^2$, where $\operatorname{sgn}\sigma_n = 1$ if $\sigma_n > 0$ and $\operatorname{sgn}\sigma_n = -1$ if $\sigma_n < 0$. σ_n and τ_n are the normal and shear stresses along the weld line, respectively. Note that the torque oscillates between a positive maximum in one direction and a minimum in the opposite direction with equal magnitude (i.e., the direction of torque T can be as shown or opposite to what is shown in the figure). If the maximum magnitude of the torque is 5×10^5 N-m, the maximum internal pressure expected is $p = 10$ MPa, and a factor of safety of 1.0 is expected, determine the minimum thickness required for this vessel.



2. An actuator is being developed by bonding a layer of piezoelectric material to a layer of elastic material. The radius of curvature of the actuator is a function of an applied electric field and can be modeled using beam theory and assuming plane stress. Figure 1 is a diagram of the actuator where the upper PZT layer of the beam is denoted by the subscript A, and the lower metal layer is denoted by the subscript B. The thickness of each layer is h, the width is w, the radius of curvature is ρ, and the distance from the neutral axis to the interface of the top and bottom layers is δ.

The constitutive behavior of the PZT is given by Equation (1). E_A is the elastic modulus, ε_{11} is strain, e_{311} is the piezoelectric coefficient, and E_3 is the electric field. When the electric field is applied in the X_3 direction, the piezoelectirc material contracts in the X_1 and X_2 directions.

Material A: $$\sigma_{11} = E_A \varepsilon_{11} + e_{311} E_3 \qquad (1)$$

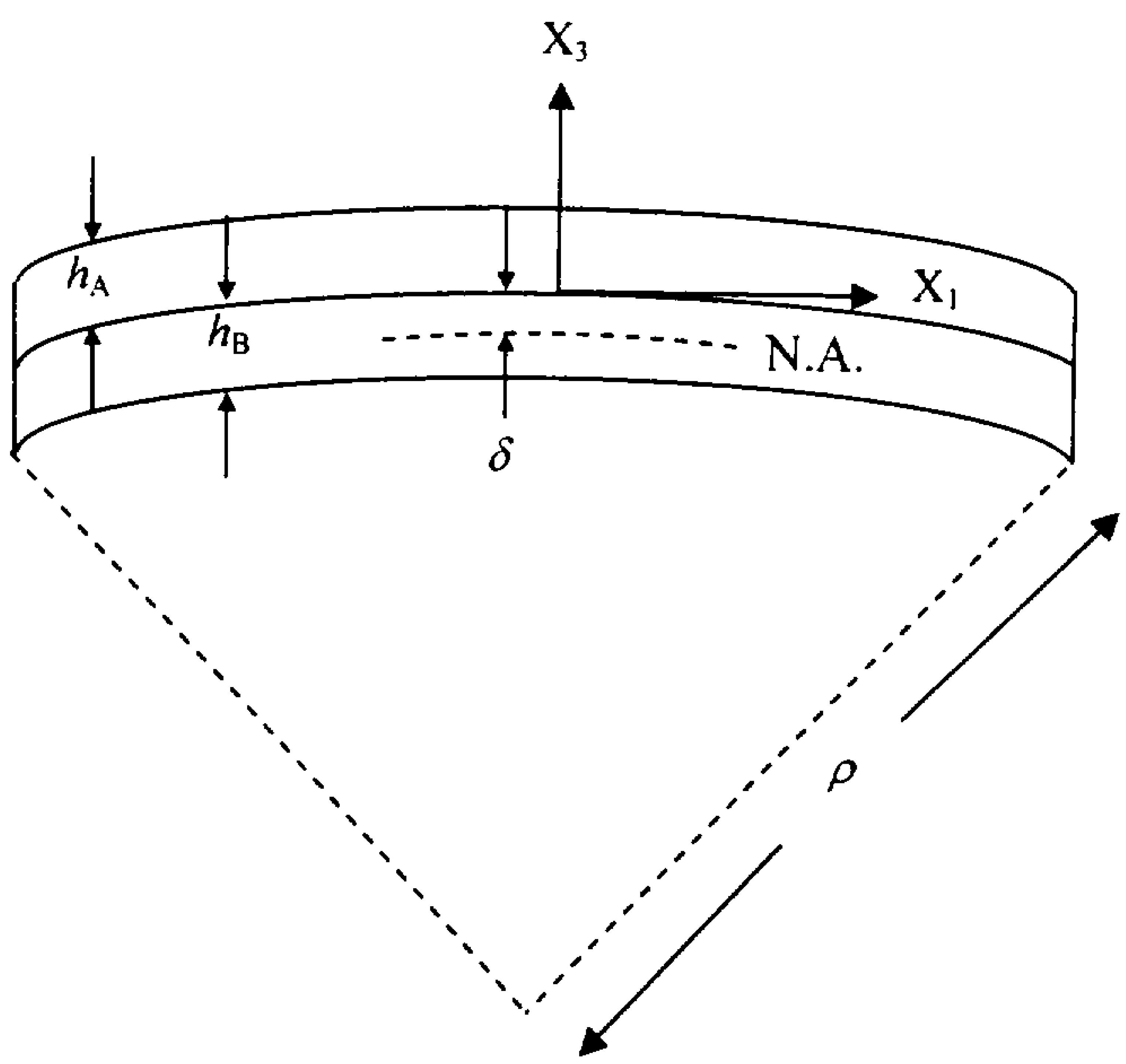
The constitutive behavior of the elastic material is given by Equation (2).

Material B: $$\sigma_{11} = E_B \varepsilon_{11} \qquad (2)$$

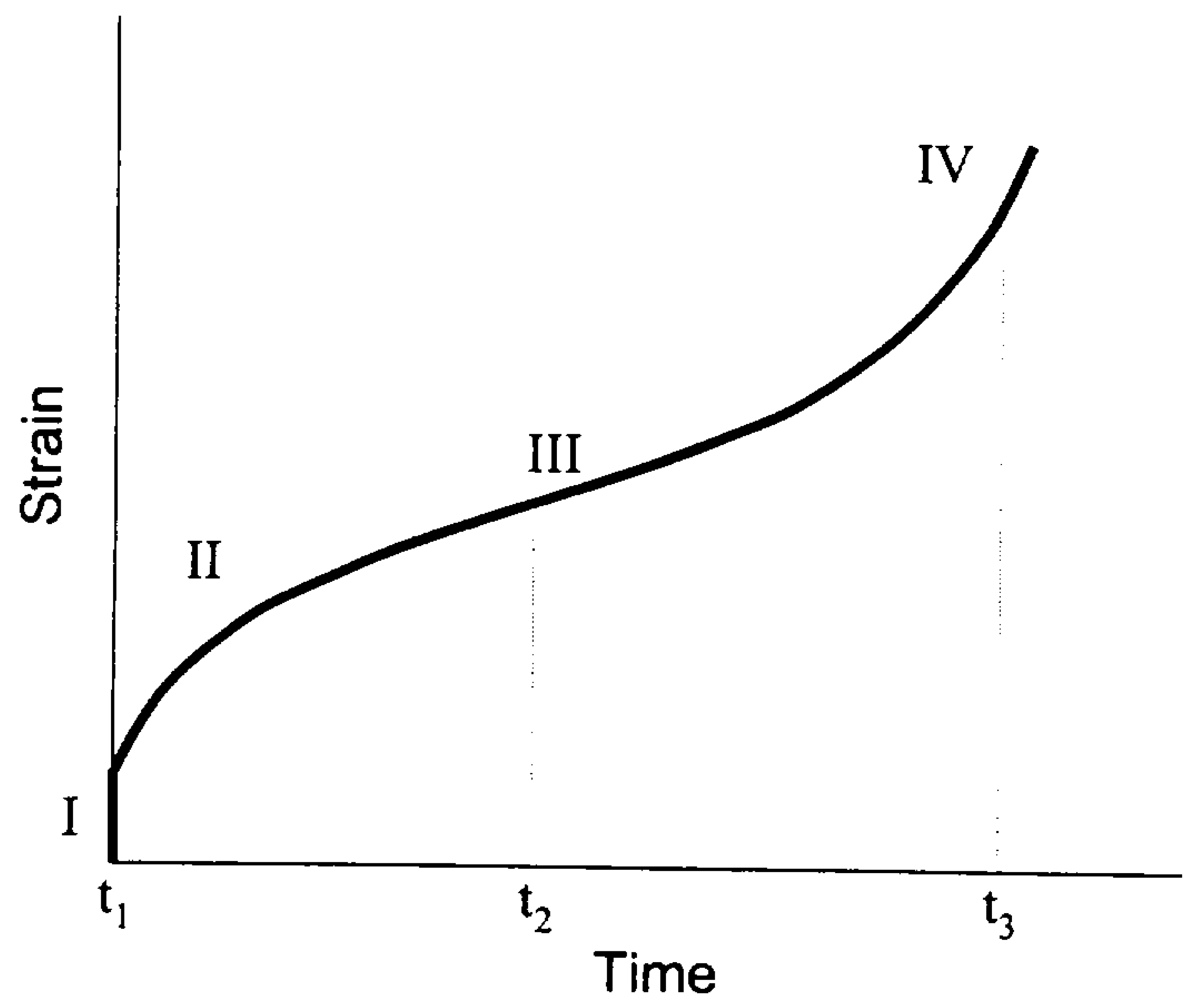
Show how you would find an equation relating the radius of curvature to an applied electric field using force and moment balance, and geometry. Perform the integration and state how the resulting equations would be solved for the radius of curvature and the distance from the neutral axis to the interface. Do not solve the equations (it is a lot of algebra).

Qualitatively explain how the results would differ if the elastic material were replaced by an elastic – perfectly plastic material.

Is there shear stress at the interface even though there are no loads applied to the beam? Give a qualitative explanation based on what you know about beam theory.



3.



Consider strain vs. time curve for a constant load uniaxial tension test at room temperature.

a. Describe what is happening in each of the four regions (I, II, III, IV) indicated.

b. Provide two alternative physical explanations for why the strain changes with time. (In other words, what could be happening microstructurally?)

c. Which of the following materials would most likely exhibit this behavior? JUSTIFY your answer.

Material	Elastic Modulus	Yield Stress	Melting Point	Thermal Expansion Coeff
Tungsten	350 GPa	200 MPa	3430 °C	$4.3 \times 10^{-6}/°C$
ASTM-A36 Steel	200 GPa	250 MPa	1480 °C	$12 \times 10^{-6}/°C$
63Sn-37Pb Solder	32 GPa	45 MPa	180 °C	$21 \times 10^{-6}/°C$

d. Consider what would happen if the load was removed at t_2. DESCRIBE the expected strain vs. time responses for a metal and for a polymer AND plot the responses above.

4. Questions on the mechanics of composite materials

a). Derive the rule-of-mixtures formula for the *longitudinal* modulus, E_l, for a unidirectional lamina. State all of the steps and sketch. (Remember that the fibers and matrix must strain the same when loaded in the longitudinal direction.)

b). Assume that the fiber modulus is 50 msi and the strength is 500 ksi, the matrix modulus is 0.5 msi and the strength is 20 ksi. The fiber volume fraction is 0.5. Calculate the ultimate strength of the unidirectional composite in the fiber direction. (msi = million pounds per square inch; ksi = thousand pounds per square inch) (Ask yourself, "Will the matrix carry any load after the fiber fails?")