GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - FALL Semester 2001

Mechanics of Materials
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

Problem 1: Consider a three-layer laminate consisting of a core of thickness h_1 and a coating layer of thickness h_2 on both sides of the core. A cross-section view of the laminate is shown in Fig. 1. The bonding between the core and coating layers is assumed perfect. The laminate is then subjected to a temperature change $\Delta T > 0$. Within this temperature range, the core layer can be considered as linear elastic with Young's modulus = E_1. The coating layers are elastic-perfect-plastic with uniaxial stress-strain relationship given by (see also Fig. 2 below),

$$\sigma = \begin{cases} E_2\varepsilon, & \varepsilon \le \varepsilon_y; \\ E_2\varepsilon_y, & \varepsilon > \varepsilon_y. \end{cases}$$

The in-plane coefficients of thermal expansion (CTE) for the core and coating layers are α_1 and α_2, respectively; and $\Delta\alpha = \alpha_1 - \alpha_2 > 0$. It is assumed that the thermomechanical properties (e.g., CTE, moduli) for both materials do not change within the temperature range considered. Furthermore, the thickness values (h_1 and h_2) are much smaller than the size of the laminate so that one-dimensional analysis is sufficient.

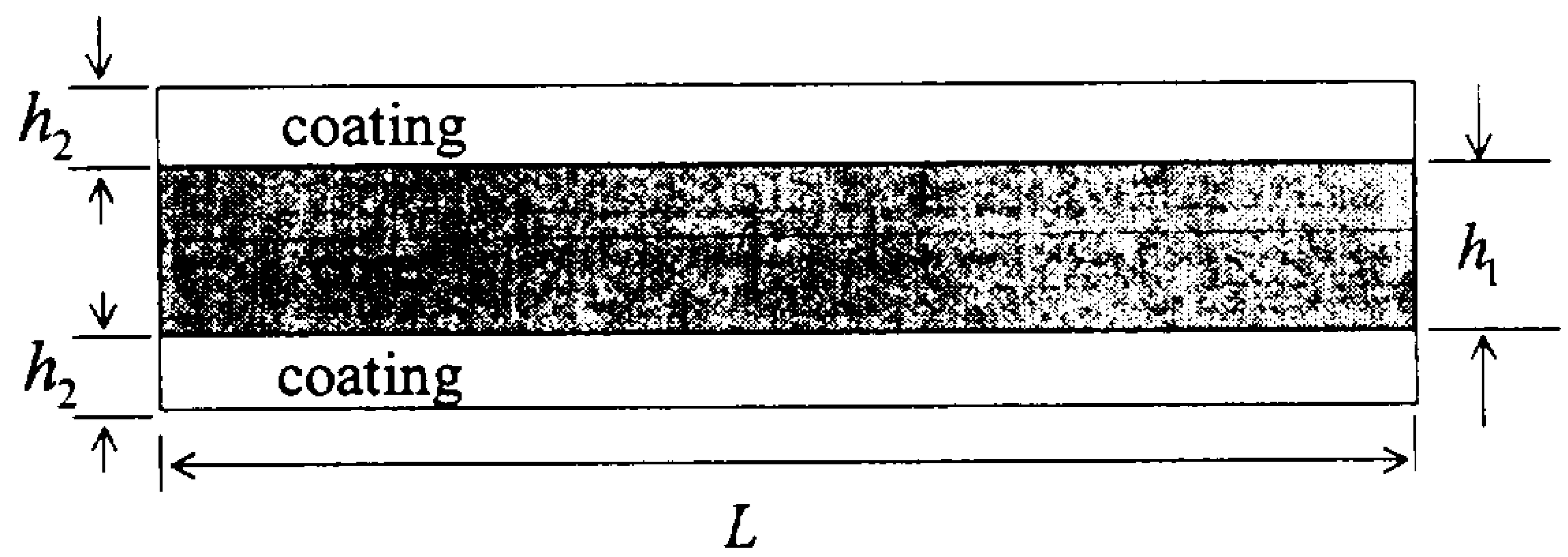


Fig. 1 for Problem 1: Cross-section of the laminate

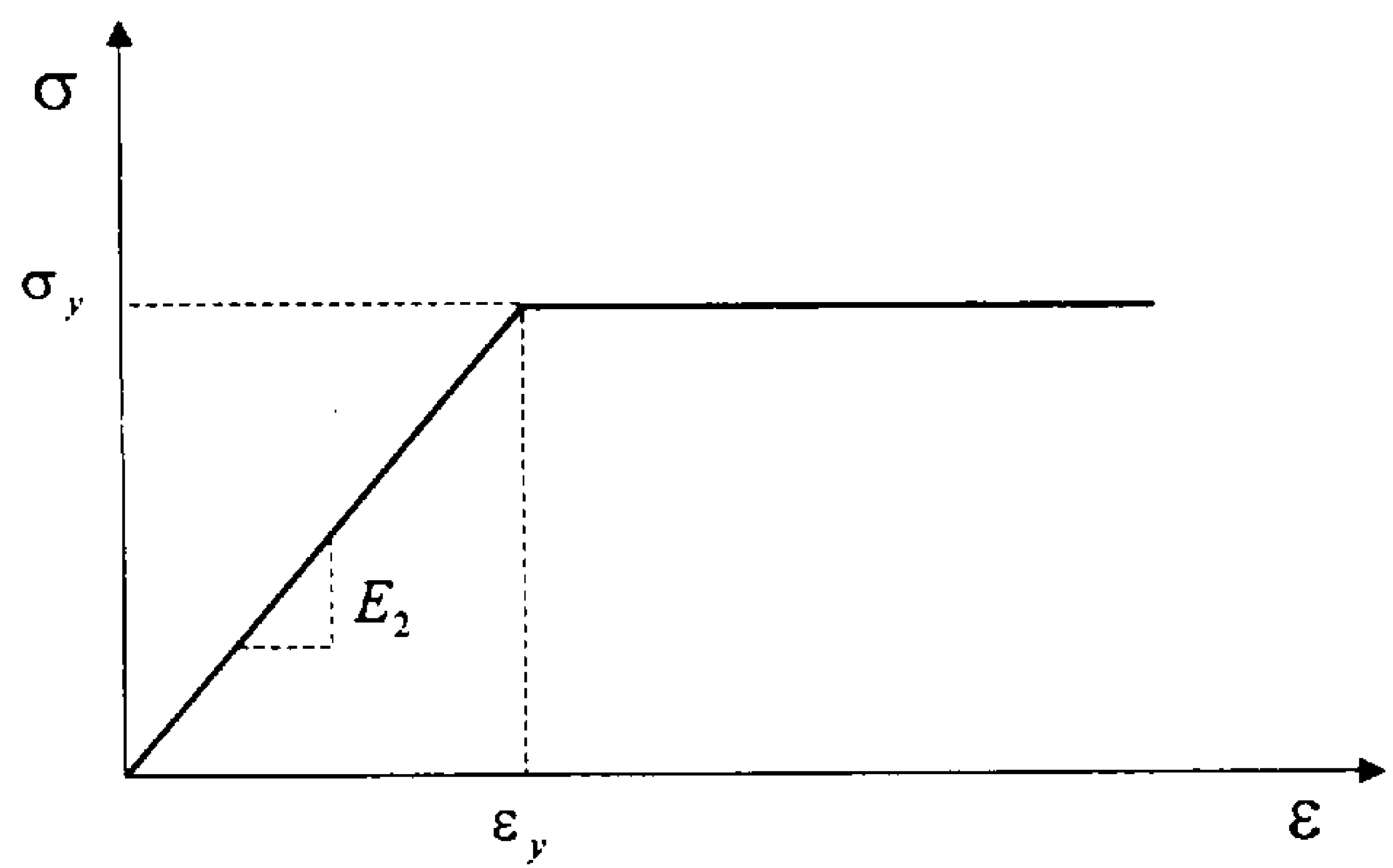


Fig. 2 for Problem 1: Stress-strain curve of the coating layers

Please find:

a) The temperature change ΔT_c that corresponds to the onset of plastic deformation in the coating layers; and

b) The total in-plane strain of the composite laminate for both $\Delta T \le \Delta T_c$ and $\Delta T > \Delta T_c$.

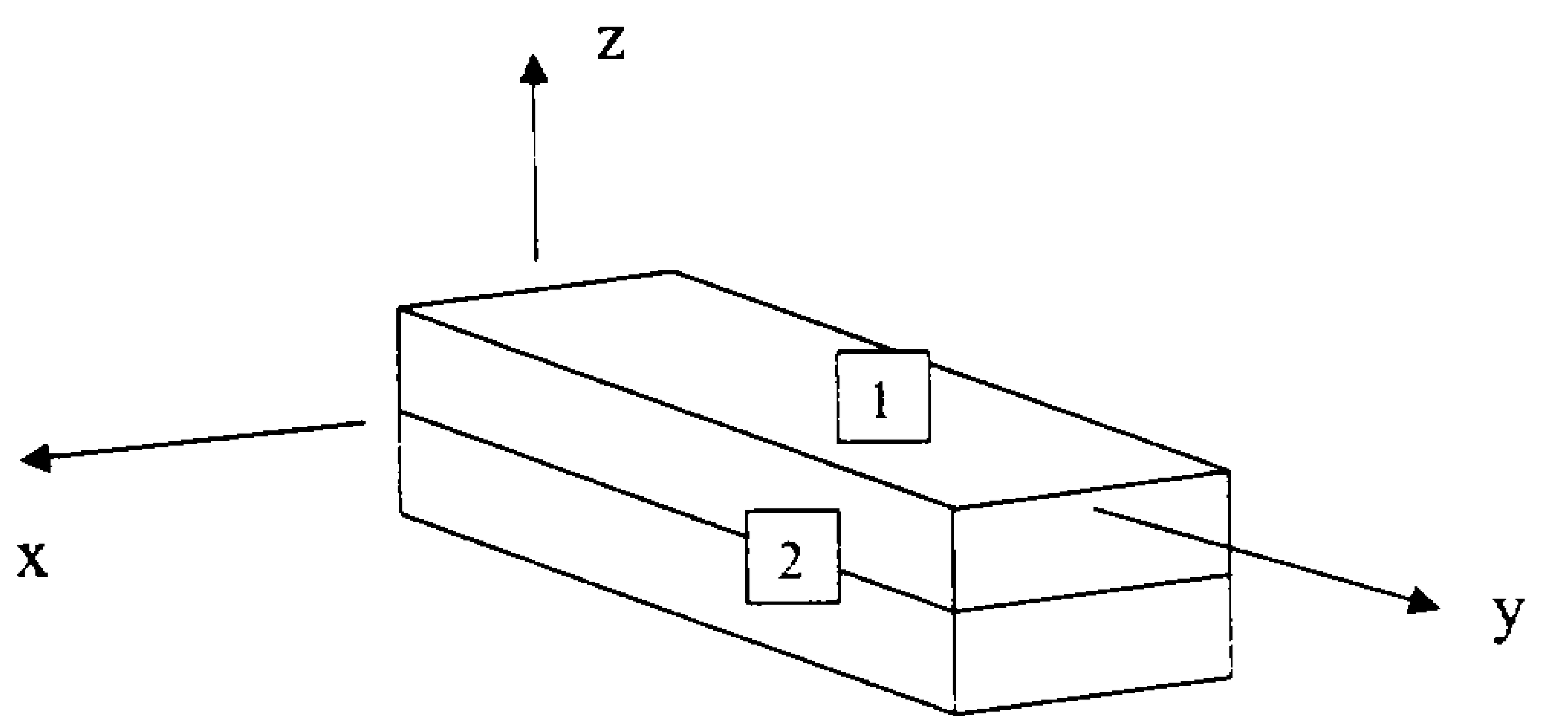
Problem 2: The bi-material beam below is subjected to pure bending about the x-axis as shown. Material 1 has graded elastic properties, while Material 2 has a constant Young's modulus.

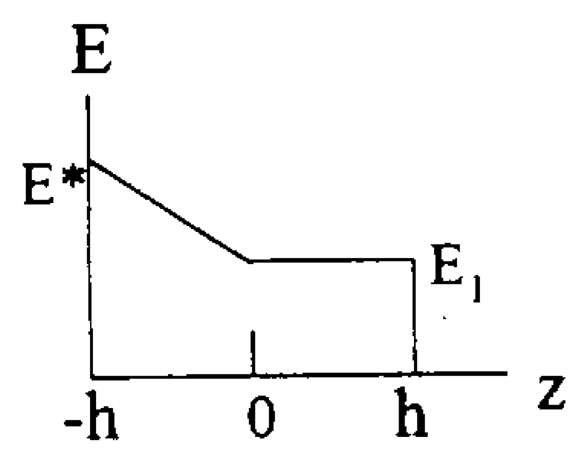
a) For a perfectly bonded interface between Materials 1 and 2, state displacement and traction continuity conditions at the interface in terms of x, y and z components of displacement and traction.

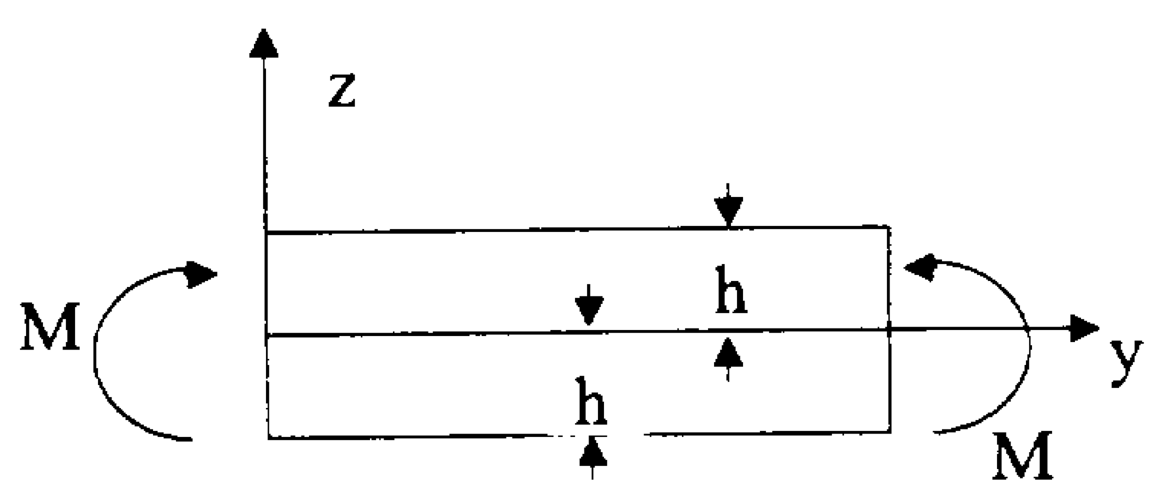
b) If the beam is linear elastic with elastic properties shown in the plot below (they are position dependent) and does not yield (i.e., yield stresses σ_{y1} and σ_{y2} are both very high), derive the stress distribution through the cross section of the bi-material beam (as a function of z) and plot it.

c) What further information is required to assess where yielding would first occur? Be precise.

d) If the interface is not bonded but instead has frictional contact between the two layers, allowing for relative sliding, please state all permissible conditions on interface traction and displacements on both sides of the interface.







Figures for Problem 2

Problem 3: Three long beams of length L, modulus of elasticity E, shear modulus G, moment of inertia I, and cross-sectional area A are welded together in the configuration shown. The structure is subjected to a load P applied at point A. The weight of the beams may be neglected. List any other simplifying assumptions that you make in your analysis. Using the energy method,

a) Find an expression for the vertical (y-direction) displacement of point A.
b) Find an expression for the horizontal (x-direction) displacement of point B.
c) What assumptions have you made in your analysis?
d) If the support at point C was changed from a pin to a built-in end, thus removing section CD from the structure, would you expect the displacements at points A and B to increase, decrease, or stay the same due to application of the same load P? (No calculations necessary but explain your answer)

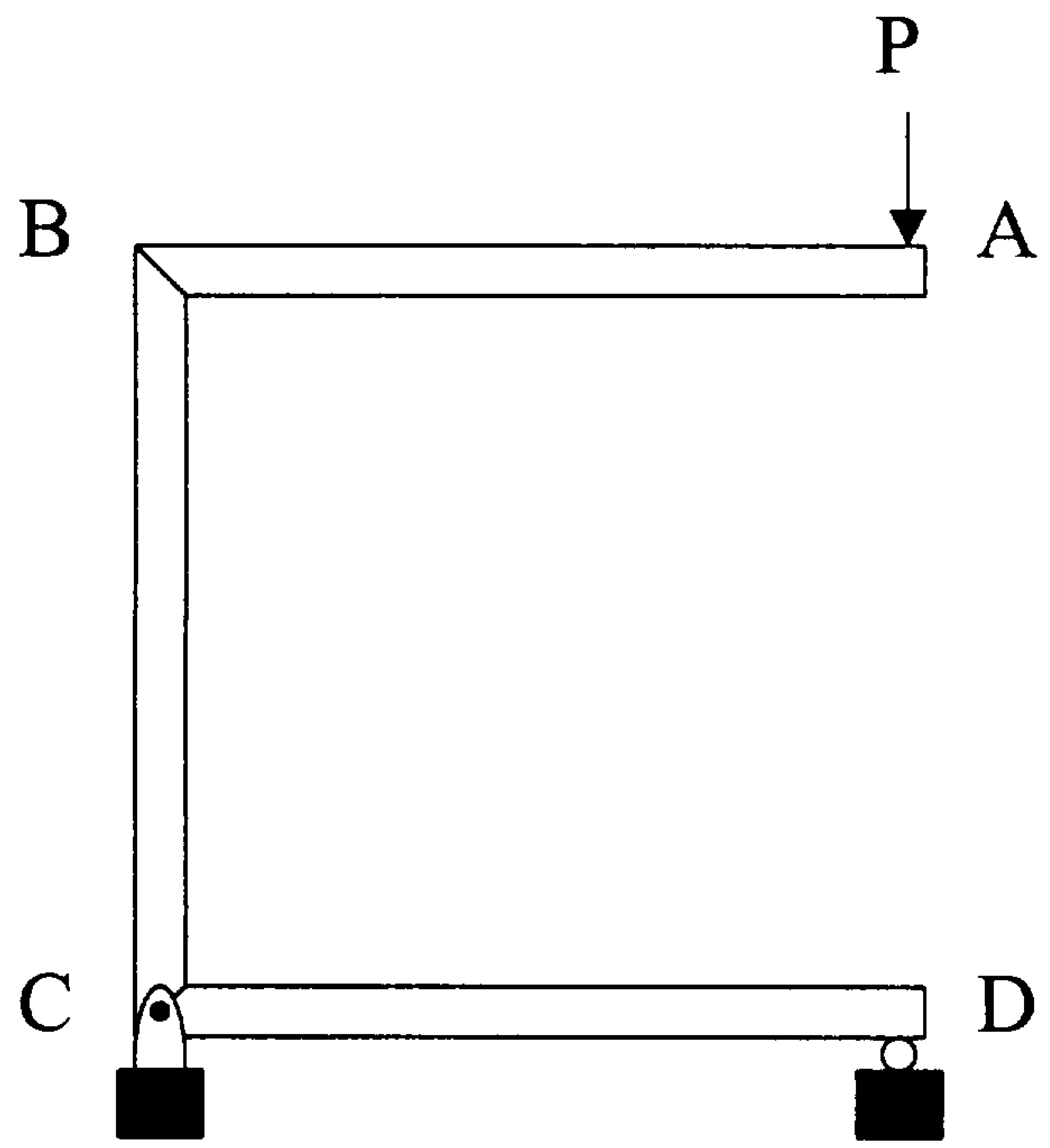


Figure for Problem 3

Problem 4: Problem on Plasticity and Fracture:

A beam is placed in four point bending as shown in the figure below. The yield strength of the material is 100 MPa. The Young's modulus is 150 GPa. (Neglect stress concentrations under the loading points in the following discussion.)

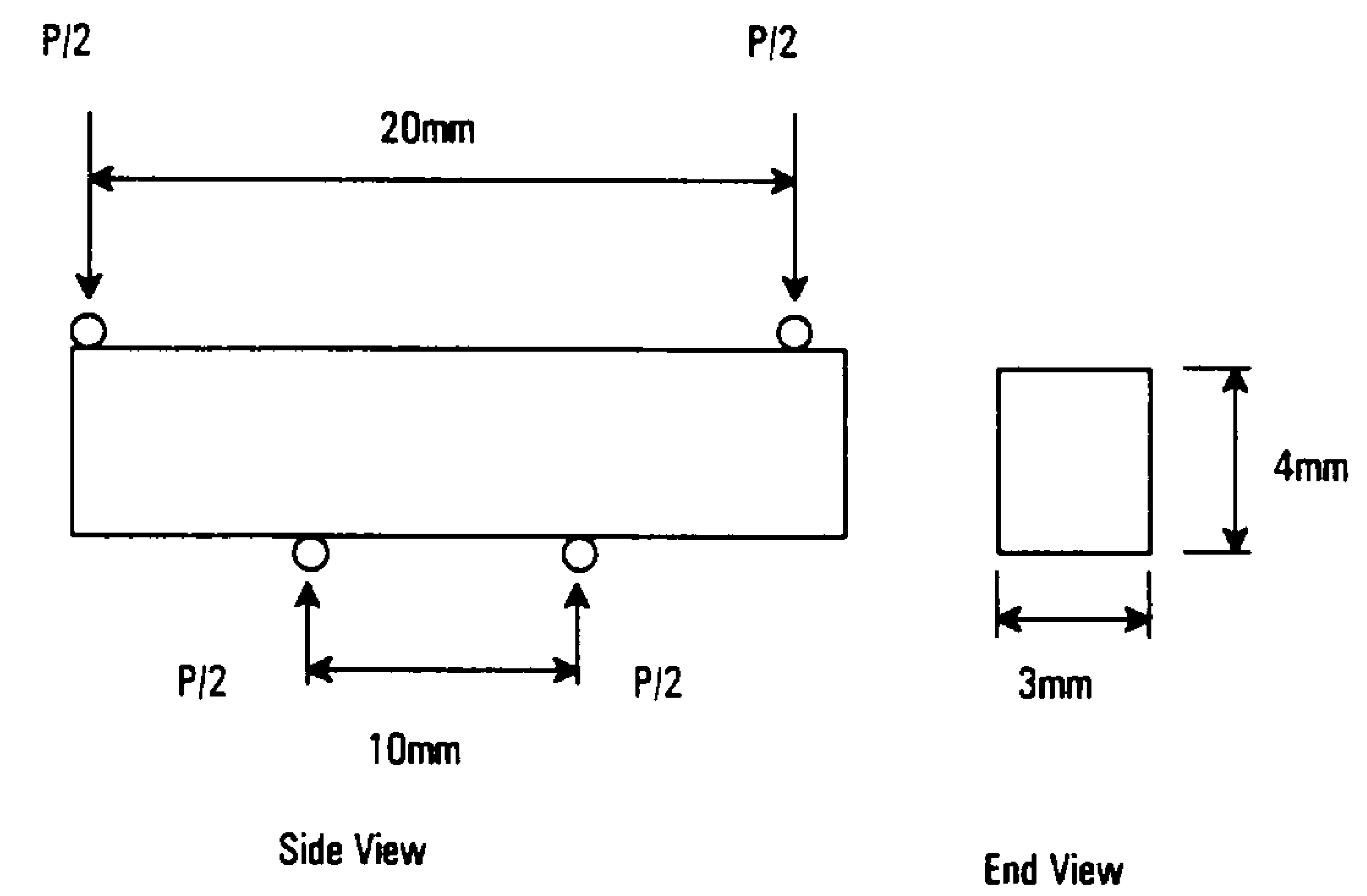


Figure for Problem 4

a) If the material is elastic – perfectly plastic, sketch the uniaxial stress-strain curve;

b) If the beam has a through crack of depth "a" on the tensile side and the material has a fracture toughness of 2 $MPam^{1/2}$, what is the critical crack size below which the beam will fail by yielding and above which it will fail by fracture? (Assume no toughening occurs).

Problem 5: The torsion rod shown below has a conical section (1) and a cylindrical section (2). It is attached to the wall at the large end of the conical section. The dimensions are indicated on the schematic.

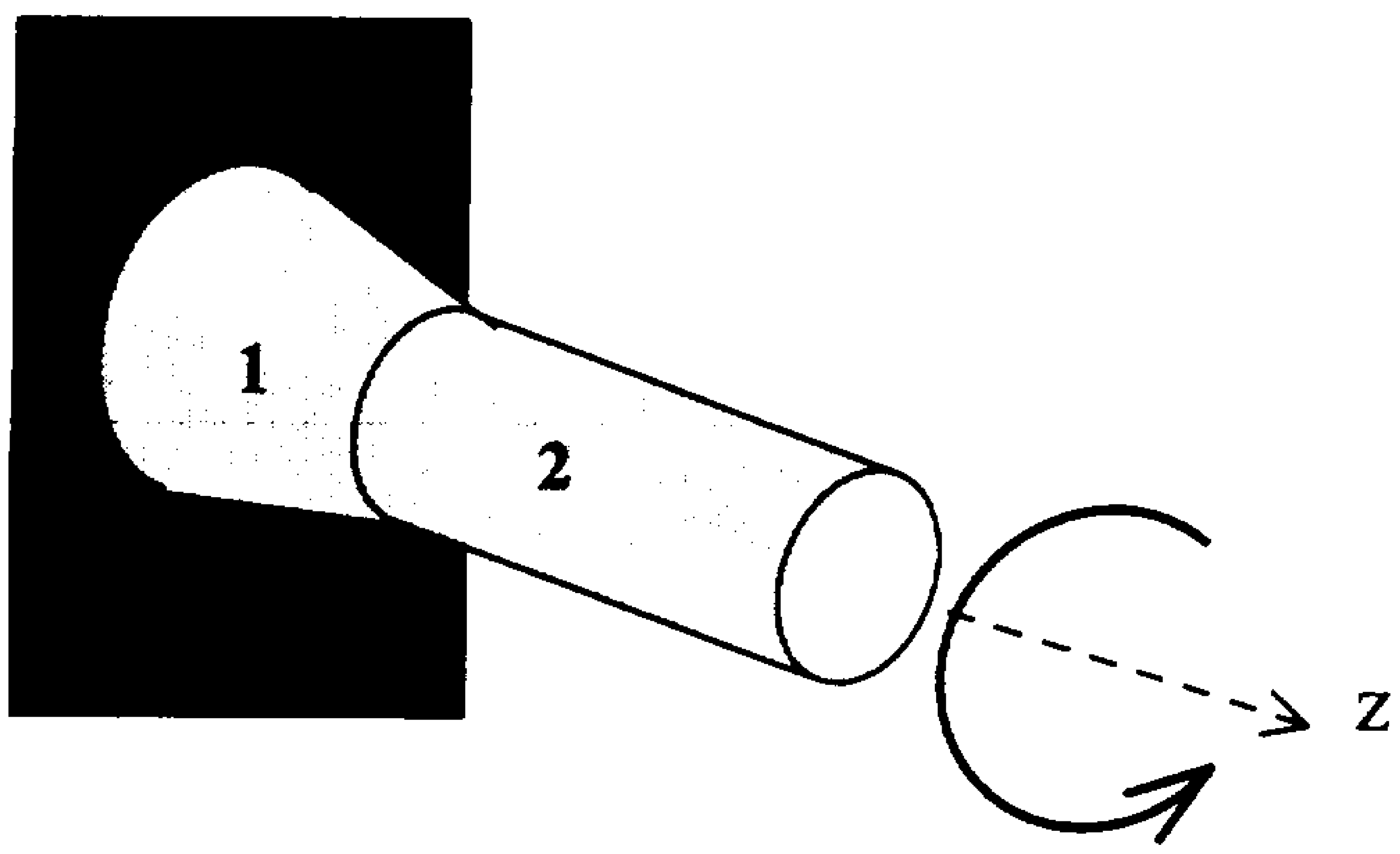
$L_2 = 2\,L_1$. $D_1 = \sqrt{2}\,D_2$

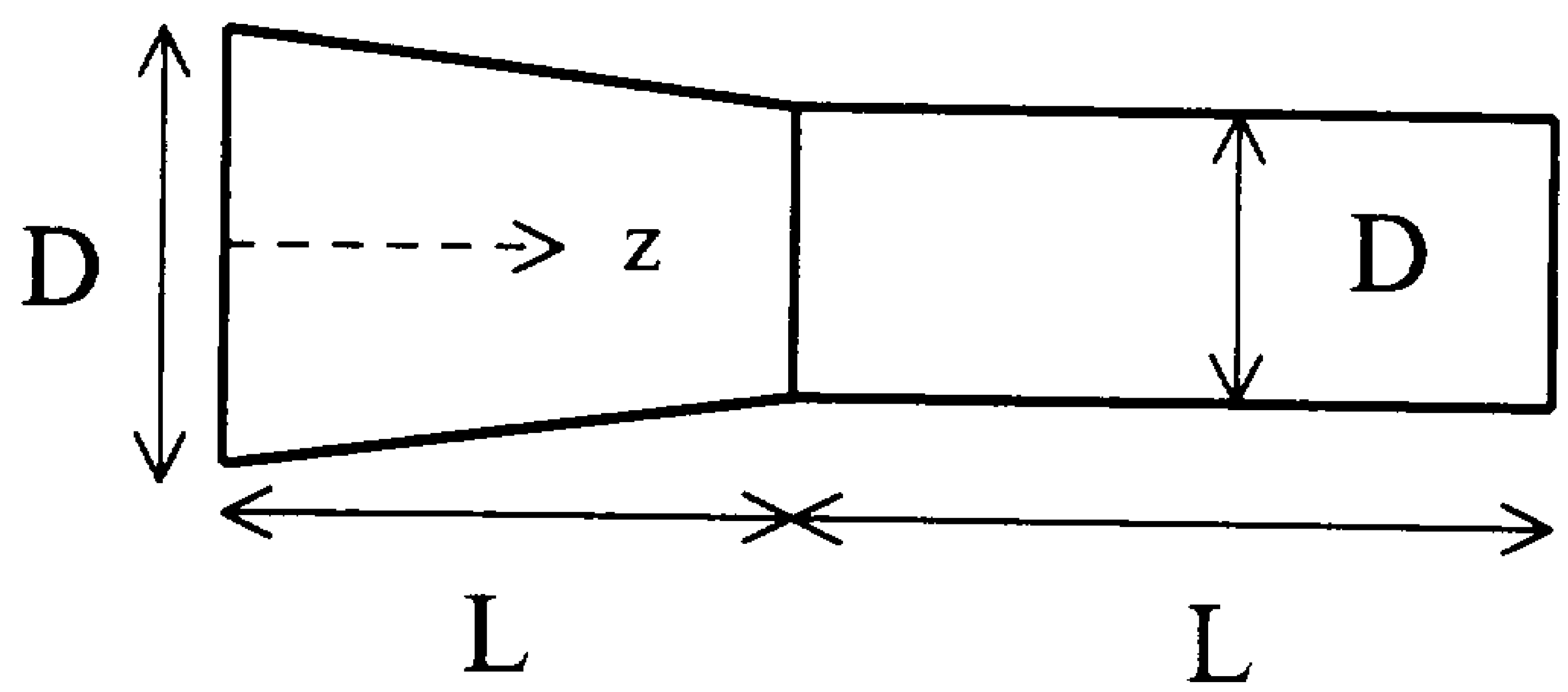
The elastic-perfectly plastic material has a shear modulus G and a yield strain γ_C.

a) If the torque is increased to the point where plastic deformation is about to occur, plot the maximum shear stress in the rod as a function of the z coordinate (z=0 at the wall).

b) Find an expression for the minimum torque T which causes the entire *cylindrical* section (2) to yield.

c) Sketch the *stress* distribution *at the wall* for that torque.





(side view)