GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff

School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Semester 1999

Mechanics of Material

EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

Problem I

(a) Give three examples of anisotropic materials.

(b) Consider a composite with all of the fibers aligned in the same direction. A schematic of the composite is shown in Figure 1 below. The anisotropic Hooke's law can be written in matrix form as

$$\begin{bmatrix}\gamma_1\\ \gamma_2\\ \gamma_3\\ \gamma_4\\ \gamma_5\\ \gamma_6\end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16}\\ s_{21} & s_{22} & s_{23} & s_{24} & s_{25} & s_{26}\\ s_{31} & s_{32} & s_{33} & s_{34} & s_{35} & s_{36}\\ s_{41} & s_{42} & s_{43} & s_{44} & s_{45} & s_{46}\\ s_{51} & s_{52} & s_{53} & s_{54} & s_{55} & s_{56}\\ s_{61} & s_{62} & s_{63} & s_{64} & s_{65} & s_{66}\end{bmatrix} \begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_4\\ \sigma_5\\ \sigma_6\end{bmatrix}$$

where

$$\begin{bmatrix}\gamma_1\\ \gamma_2\\ \gamma_3\\ \gamma_4\\ \gamma_5\\ \gamma_6\end{bmatrix} = \begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{23}\\ 2\varepsilon_{31}\\ 2\varepsilon_{12}\end{bmatrix} \quad \text{and} \quad \begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_4\\ \sigma_5\\ \sigma_6\end{bmatrix} = \begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{23}\\ \sigma_{31}\\ \sigma_{12}\end{bmatrix}$$

What coefficients of the compliance matrix are zero and what coefficients are equivalent? (Hint: You may use the knowledge that the matrix is symmetric).

No credit will be given without a brief example stating why the coefficients are equivalent or zero. Put your final results in matrix form.

(c) For the same composite, develop an expression for the Young's modulus parallel to the fibers in terms of E_f, E_m, and f. E_f is the Young's modulus of the fibers, E_m is the Young's modulus of the matrix material, and f is the volume fraction of the fibers (same as the area fraction).

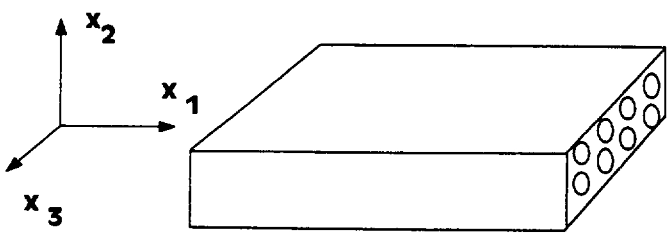


Figure 1. A schematic of a fiber reinforced composite. All of the fibers align with the x_1 direction.

Problem II

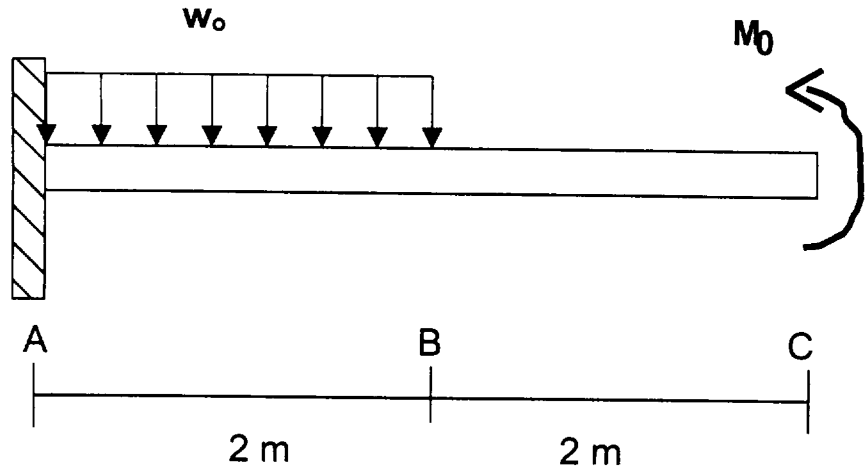
For isotropic, linear elastic materials, the Young's modulus E, Poisson's ratio ν, and the shear modulus G are related through $G = \dfrac{E}{2(1+\nu)}$. Use your knowledge of stress and strain analysis and the generalized Hooke's law to show that this relation is true.

Problem III

A short thin-walled pipe made of polyvinylchloride (PVC) that is constrained by end fittings is subjected to an internal pressure of 700 kPa at 20°C. If the internal diameter is 200 mm and a tensile stress of 17.5 MPa is not to be exceeded, (a) Determine a suitable wall thickness for the pipe. (b) What will be the increase in diameter after 1000 hours? The creep strains at 1000 hours for different tensile stresses applied to PVC at 20°C are given in the table below. The mean creep contraction ratio (ν_t) is 0.45.

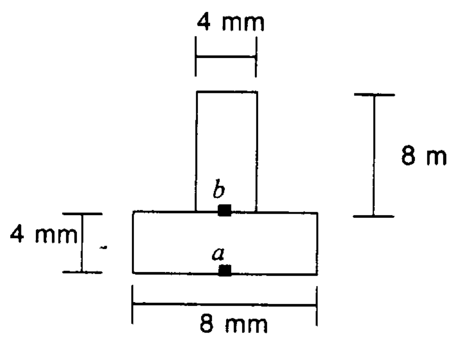
σ (MPa)	ε^{cr} (%)
6.9	0.2
13.8	0.48
20.7	0.92
27.6	1.72
34.5	3.38

Problem IV



M_0 = 8 N-m, w_o = 10 N/m

Beam Cross-Section



The beam shown above supports a point moment $\mathbf{M_o}$ and a distributed load $\mathbf{w_o}$.

Clearly state all **assumptions.**

1. Draw the **shear force** and **bending moment** diagrams (label all points/curves).

2. Find a transverse cross-section that has the largest **normal** stress in the beam and calculate values of the normal stress at points *a* & *b* on that cross-section.

3. Find a transverse cross-section that has the largest **shear** stress in the beam and calculate values of the shear stress at points *a* & *b* on that

cross-section.

4. Find the deflection curve $v(x)$ in terms of EI for the **AB interval** only.