Work 3 out 4 questions.

1. The figure shows a simple mechanical system with mass m_1, dampers b_1, b_2, springs k_1, k_2, force f, and displacements x_1, x_2.

a) Determine a set of governing differential equations in terms of input $f(t)$ and outputs $x_1(t)$, $x_2(t)$.

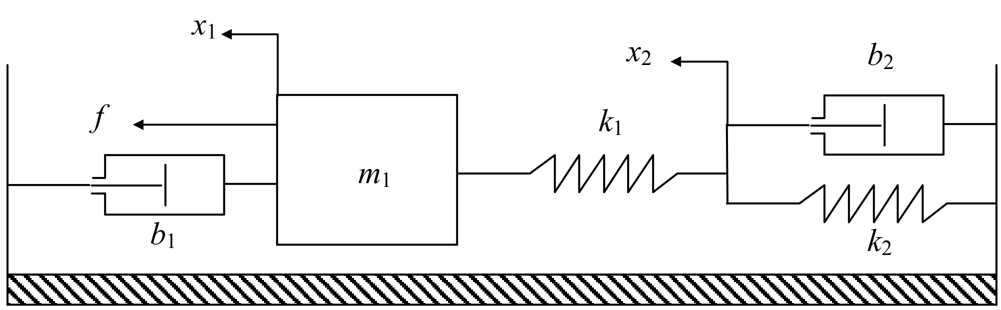
b) Draw a block diagram showing both $X_1(s)$ and $X_2(s)$ explicitly.

c) Determine the transfer function $X_1(s)/ F(s)$.

d) Redraw the block diagram in b) with the following:

   i. Show the velocity $V_1(s)$ explicitly

   ii. No block may contain an s in the numerator (i.e. you cannot differentiate $X_1(s)$ to get $V_1(s)$).



2. Consider a unity feedback system whose open-loop (feedforward) portion is a proportional controller (P controller) followed by a third order system

$$G(s) = \frac{3s^2 + s}{s^3 + bs + a}.$$

Here a and b are constant system parameters. Their exact values are unknown except that they are in the ranges

$$2 < a < 6,\ 1 < b < 3.$$

Determine the range of the proportional gain which *guarantees* the stability of the closed-loop system.

3.

Aerial Lift in Atlantic Station.

Aerial lifts, like the one shown in Figure 1 are used to raise workers to high heights. The machine is a telescoping robotic arm that is driven by operators riding in a basket at the end of the arm. The machine rotates its base about a vertical axis; it changes the elevation angle of its arm; it telescopes its arm; and the basket can make small adjustment motions. If the basket oscillates, then it is difficult for the person to work safely and efficiently.

1) Create a simple model that can predict the basket oscillation when the machine is moved by the operator. The operator moves the machine by pressing control levers. You can assume that these inputs result in step forces acting on the machine. The simple model can be linearized around a set of operating conditions such as the length of the telescoping arm and the mass in the basket.

2) Sketch the time response of the basket when the operator i) presses a lever and holds it down to rotate the machine about its base, ii) presses a rotation lever for 2 seconds and then releases it, iii) extends the arm 50% more and then rotates it for 2 seconds.

3) Develop a simple control system to decrease the basket oscillation. You can assume that the bending angle of the telescoping arm can be measured.

4) Sketch the root-locus of the system when one of your controller gains is varied. What is the effect on the root locus when the basket mass is doubled?

4.

Consider a stable dynamic system represented the following transfer function:

$$G(s) = \frac{cs + d}{s^2 + as + b} \quad (a, b, c, d \text{ :Constants})$$

The following properties are known about $G(s)$.

- The unit step response of $G(s)$ becomes 0 as $t \to \infty$.
- The unit ramp response of $G(s)$ becomes 1 as $t \to \infty$.
- When $x(t) = \sin t$ is given as an input to $G(s)$, the steady-state output is also $y(t) = \sin t$.

(1) Obtain parameters a, b, c, d.

(2) Use the parameters obtained in (1). Sketch the bode diagram of $G(s)$.