

**ME Ph.D. Qualifying Examination — System Dynamics and Controls, Spring 2009**

Choose 3 of the following 4 questions to answer.

**Problem 1**

Consider a unity-feedback PID controlled system with the open-loop transfer function:

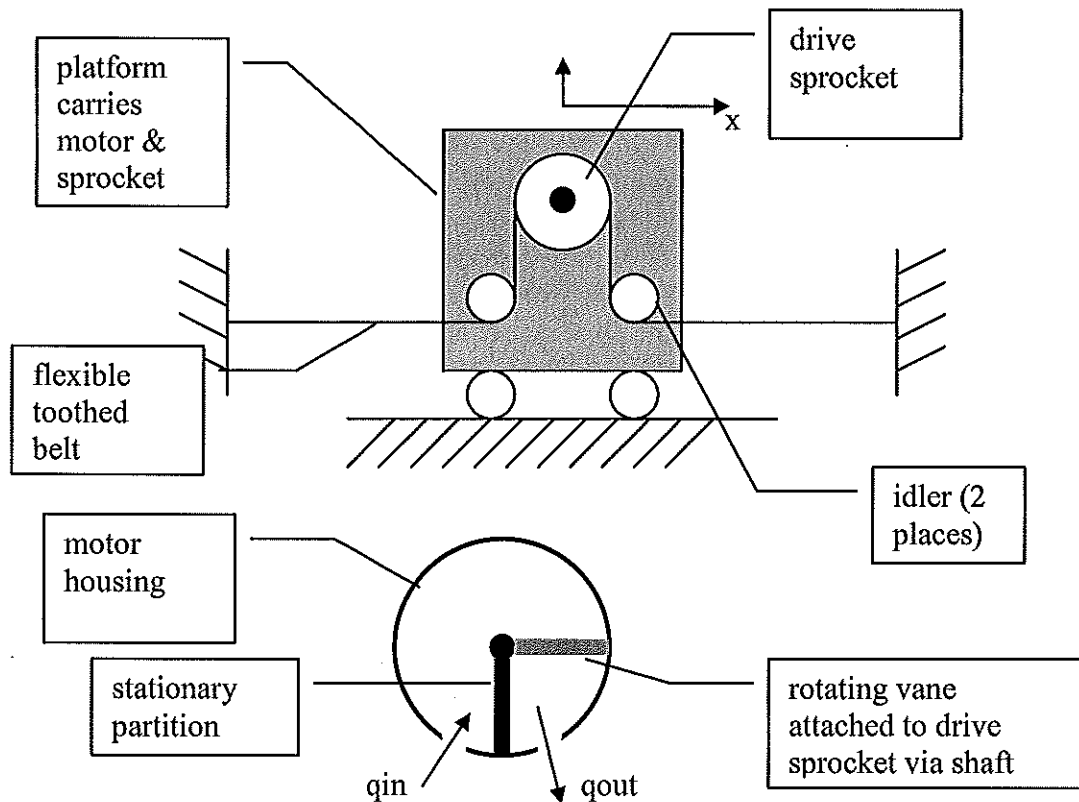
$$G(s) = \frac{K}{s(1+0.1s)(1+0.2s)}.$$

Starting with a simple proportional controller with the gain  $K_p=1$ , an engineer designs the  $K$  value for the mechanical hardware so that the ramp-error constant of the system is 100. He then realizes that the closed-loop system (with only  $K_p=1$ ) is unstable since the pair of complex poles are  $3.8 \pm j14.4$ .

- (a) In an attempt to improve the system with a PD controller, he introduces a derivative gain  $K_d$  while keeping  $K=100$  and  $K_p=1$  to achieve a reasonable transient response; say, the dominant complex poles  $-a \pm jb$  have a relative damping ratio of 0.7 and  $a > 7.5$ . Use a root locus plot to illustrate the effect of  $K_d$  on the closed loop poles.
- (b) Design a PID controller (using root locus OR bode diagrams) to achieve the above specification (in terms of the ramp error constant and dominant closed-loop poles). Show your approach clearly but NO detailed numerical calculation is required.

### Problem 2

Shown schematically below is an “omega drive” arrangement for moving a platform via a sprocket engaging a flexible toothed belt. The drive sprocket is powered by a vane type hydraulic actuator diagrammed separately. Flow  $q_{in}$  is the input into the actuator is controlled by a servovalve that is not part of the system you are to model. The total length of the belt is  $L$ , the drive sprocket has radius  $r_s$  and the inner radius of the motor is  $r_p$  and its depth out of the page is  $d$ . You can assume negligible flow losses. The compliance of the belt per meter of length is  $c_b$ .  $q_{in}$  is the volumetric rate of flow into the motor as shown, with positive  $q_{in}$  resulting in a positive change in platform position  $x$ .  $x$  is measured from the midpoint position as shown. Total mass of the platform is  $M$ .



- Without deriving equations of motion, determine the order of the system above and explain why this is the order you expect (without deriving equations).
- Derive the nonlinear equations of motion of the system as defined above. The equations should yield  $x$  as the output with  $q_{in}$  as the input. State any assumptions beyond those made above.
- Linearize these equations of motion about zero velocity at  $x = L/5$ .
- A more detailed model could include the compressibility of the fluid. It is determined that with a change of pressure of  $dp$  the volume of fluid decreases by  $dv/dp = c_f$ . Now what are the dynamic equation(s) of the system?

**Problem 3**

Consider a unity-feedback control system whose feedforward portion

$$G(s) = \frac{K(s+1)}{(s-1)(s+2)}$$

Use the Nyquist stability criterion to determine the range of  $K$  such that the closed-loop system is stable.

#### Problem 4

With ever-increasing fuel costs and laziness, there is an emerging market for personal human transporters. Perhaps the most widely-known such product is the Segway, shown in Figure 1. Unfortunately, the Segway falls over without active feedback control. This property makes the device dangerous – especially for those riders who do not have good jumping, landing, and falling skills. Therefore, numerous other human transporters have been developed that have 3 or more wheels, such as the Eco-Stand Scooter shown in Figure 2.



Figure 1: Segway Human Transporter.



Figure 2: Eco-Stand-Up Scooter  
<http://www.ilhumantransporter.com/>

- 1) Using simple models and equations, explain why the Segway naturally falls over and the Eco-Scooter does not (assume no feedback control for either machine and do NOT include a human passenger.)
- 2) Now, add a human operator to the system and explain what effect they have on the stability of both machines.
- 3) Develop a simple controller that would stabilize the Segway.
- 4) What type of sensors, and how many, would be required for your controller? What noise sources, or problems would you expect with your sensors?
- 5) What tests would you run and what performance data would you collect to compare the Segway and the Eco-Scooter?
- 6) Sketch the time responses of both machines to a “Go Forward” command from the operator.