Special Instructions:

Please choose 3 out of the four problems given in this exam and clearly indicate which 3 problems you want to be graded. If you fail to clearly identify your choices or choose to do all 4 problems, the first three problems will be graded.

Problem 1:

Figure 1(a) shows a schematic of an air bearing, which is used in pair as shown in Figure 1(b) to regulate the armature (mass *m*) in the *x* direction about its center. For a given geometry and constant supply pressure p_s, the mass flow-rte of the supply and exhaust air, *q* and q_o, are given respectively by

$$q = a\sqrt{p(p_s - p)} \text{ and } q_o = b(p - p_a)h$$

where *a* and *b* are constants; *p* is the average pressure in the air pocket; *h* is the dimension of the air gap; and p_a is the atmospheric pressure. Assume that the effective area of the air bearing is A_e, and that the mass of the air stored in the air pocket is given by $M = cp + dh$ where *c* and *d* are constants. Derive a linearized dynamic equation to describe the motion of the armature, *h(t)* and give the criteria for the stability of the air bearing regulator.

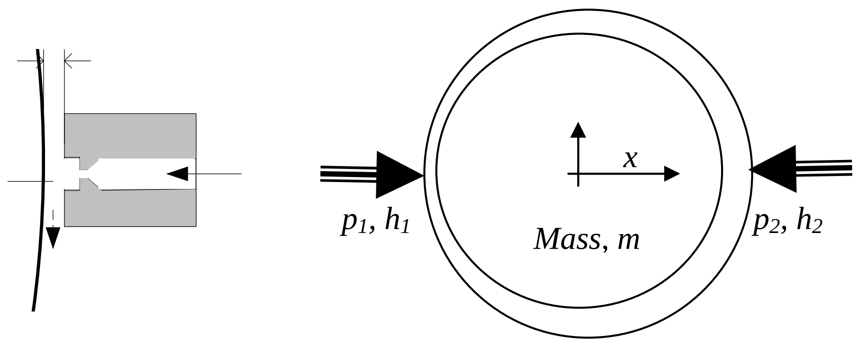
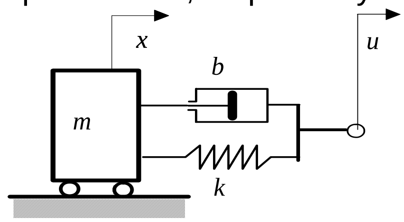


Figure 1(a) Figure 1(b)

Problem 2:

Consider the mass-spring-damper system shown in the figure below where u and x denote the input and output displacements, respectively:



a) Show that the input-output transfer function of the system (X/U) after normalizing the time variable can be expressed by

$$G(s) = \frac{2\zeta s + 1}{s^2 + 2\zeta s + 1}$$

Determine the time normalization factor and the damping ratio ζ in terms of the system parameters (m,b, and k).

b) Derive an expression for the unit impulse and step responses of the system in terms of ζ.

c) Derive a neat expression for the maximum overshoot percentage of the step response of the system assuming $\sqrt{2}/2 \le \zeta \le 1$. In particular, does the response exhibit any overshoot when the system is critically damped ($\zeta = 1$)? Sketch the corresponding unit step response.

d) Determine the phase margin of G(s) as a function of ζ and explain its significance.

Laplace Transform Table

	$f(t)$	$F(s) = \mathcal{L}[f(t)]$
1	$\delta(t)$	1
2	$\mathbf{1}(t)$	$1/s$
3	t	$1/s^2$
4	t^n	$n!/s^{n+1}$
5	e^{-at}	$1/(s+a)$
7	$t^n e^{-at}$	$n!/(s+a)^{n+1}$
8	$\sin \omega t$	$\omega/[s^2+\omega^2]$
9	$\cos \omega t$	$s/[s^2+\omega^2]$
10	$e^{-at} \sin \omega t$	$\omega/[(s+a)^2+\omega^2]$
11	$e^{-at} \cos \omega t$	$(s+a)/[(s+a)^2+\omega^2]$

Problem 3:

A unity-feedback control system with a PI compensator is used to control the response of a plant described by the following transfer function:

$$G(s) = \frac{3s+1}{s^2+3s}$$

(a) Find the range of values of the integral gain for which the closed-loop system output is able to follow the reference input $r(t) = t^2$ with a steady state tracking error $e_{ss} \leq 6$.

(b) Fixing K_i at the smallest value found in part (a), find the smallest value of the proportional gain for which the closed-loop system step response is able to reach $\pm 2\%$ of its steady state value with a settling time $t_{s,2\%} \leq 1$. You may make the following simplifying assumption: the closed-loop poles whose trajectories are terminated by finite closed-loop zeros are sufficiently close to those zeros that they contribute negligibly to the transient response.

<u>Problem 4</u>:
Consider the 3rd order system:

$$G(s) = \frac{s+100}{s(s+10)(s+50)}$$

(a) Sketch the Bode Plot for G(s). Two sets of grid lines are provided below for your convenience.

(b) KG(s) is placed in the forward path of a unity, negative feedback loop, i.e. the standard design. Find the value of K that will result in 10dB gain margin.

(c) Sketch the Bode plot for KG and indicate the corresponding phase margin on the plot.

(d) What gain K will result in 0 phase margin?

(e) In addition to the dynamics previously considered, consider additionally a time delay of 0.1s added to the transfer function G(s). What is the phase margin for the gain found in part (d).