GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Semester 2001

System Dynamics & Controls
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

Problem #1

A new ride is being developed for the Stew and Spew Amusement Park. A schematic diagram of the proposed machine is shown in Figure 1. Riders are strapped into chairs that are connected to an Overhead Support through a cable. The overhead structure rotates in the θ direction, thereby swinging the riders around. Similar rides currently exist, but the proposed ride has two novel features:

1) The angular velocity of the support structure dθ/dt, will be a complicated function of time. (The current rides just have a trapezoidal velocity profile.) It is hoped that this complicated motion of the overhead support will lead to a much more interesting ride experience.

2) The Overhead Support moves in the vertical, z, direction. The proposed Tower Beam is 100 ft. tall. The Overhead Support will travel up the Tower slowly, but will be allowed to drop rapidly, giving a freefall experience.

Your job is to create a dynamic model of the proposed system so that the ride experience can be accessed and the mechanical design can be finalized.

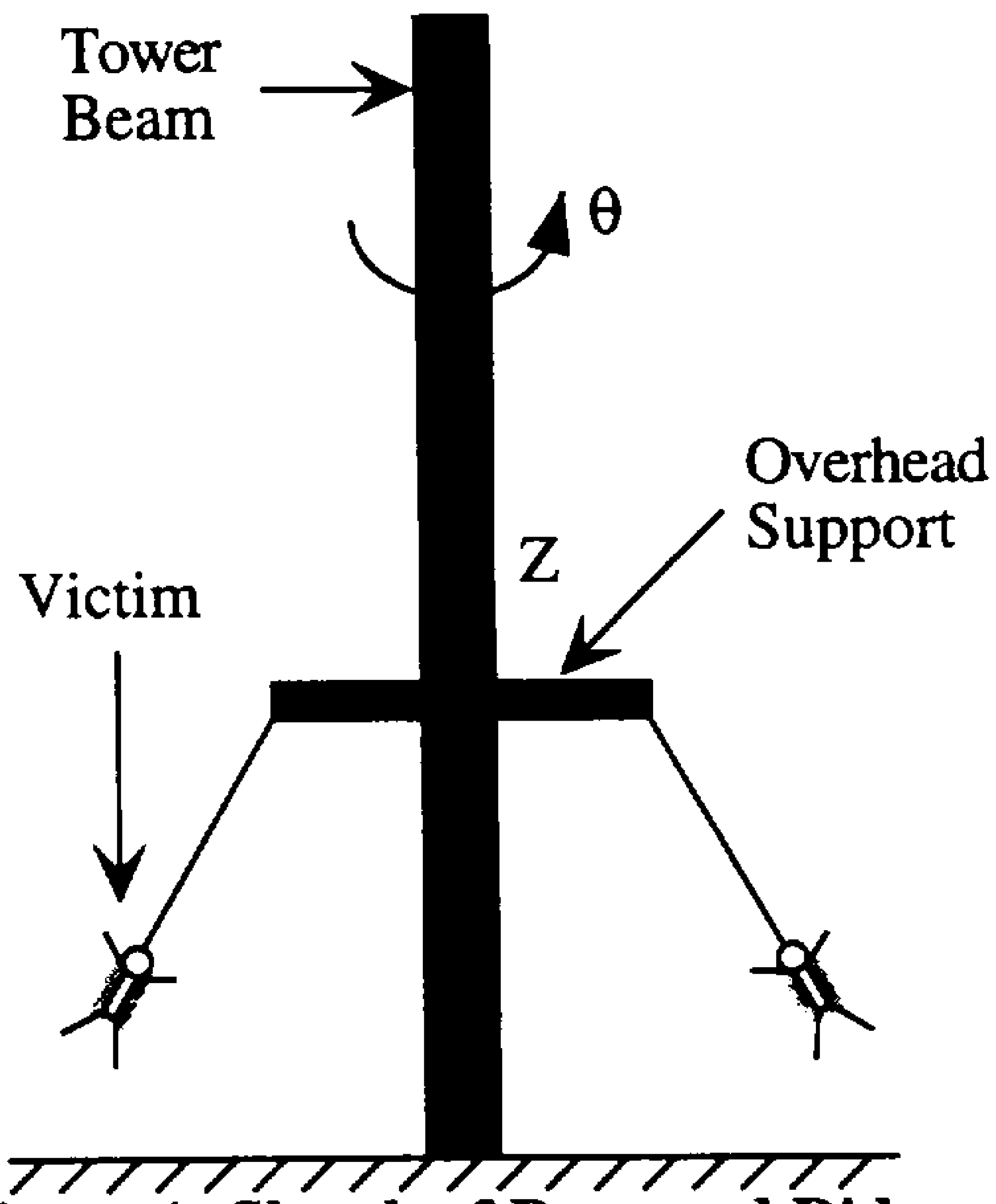


Figure 1: Sketch of Proposed Ride.

1) Make a list of the important components that should be in the model. (This is just a list of words, i.e., Overhead Support Actuator, Bird Droppings on Overheat Support, Victim's Eyelashes. Mathematical models will be required in part 2).

2) Describe how you would model each of the components. Either state clearly in words, or provide a mathematical model, like a transfer function.

3) When the Overhead Support is near the top of the tower, the dynamics might be very different than when the Overhead Support is at the bottom. How could you simplify the model for the low-altitude case?

4) Using your subcomponent models from part 2), derive a mathematical model for the low-altitude case. Assume the input is actuator torque, T, applied to the Overhead Support and the output is the θ position of one of the riders. (Comment of how you would also get the swing out angle.)

5) Describe how you would model the high-altitude situation. Just use G(s) to represent your answer to part 4.

6) Suppose that all of the chairs are not filled with people. How will this effect the high-altitude dynamics? Sketch a time response curve for the top of the beam.

Some *possibly* useful Laplace properties are on the following page.

$f(t)$	$F(s)$
Unit impulse $\delta(t)$	1
Unit step $1(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$
$1-\cos \omega t$	$\frac{\omega^2}{s(s^2+\omega^2)}$

$$\mathcal{L}[e^{-at}f(t)] = F(s+a)$$

$$\mathcal{L}[f(t-\alpha)1(t-\alpha)] = e^{-\alpha s}F(s) \qquad \alpha \geq 0$$

2. The Bode diagram given in Figure 2(b) was obtained from the closed-loop position control system shown below, where the input and output signals of the frequency response were at the points m(t) and b(t), indicated in Figure 2(a), respectively. It was determined in a separate experiment that H(s) = 4.

Figure 2(a)

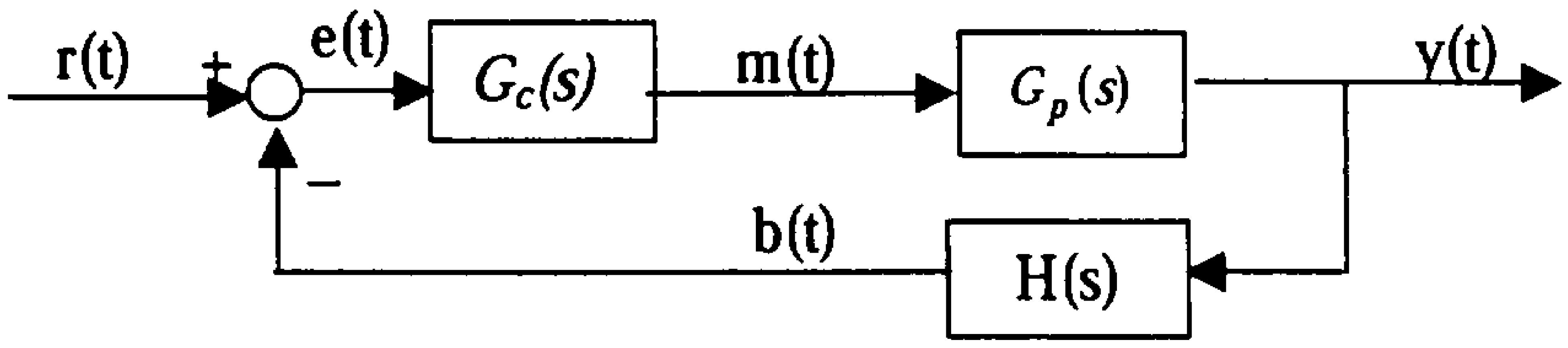
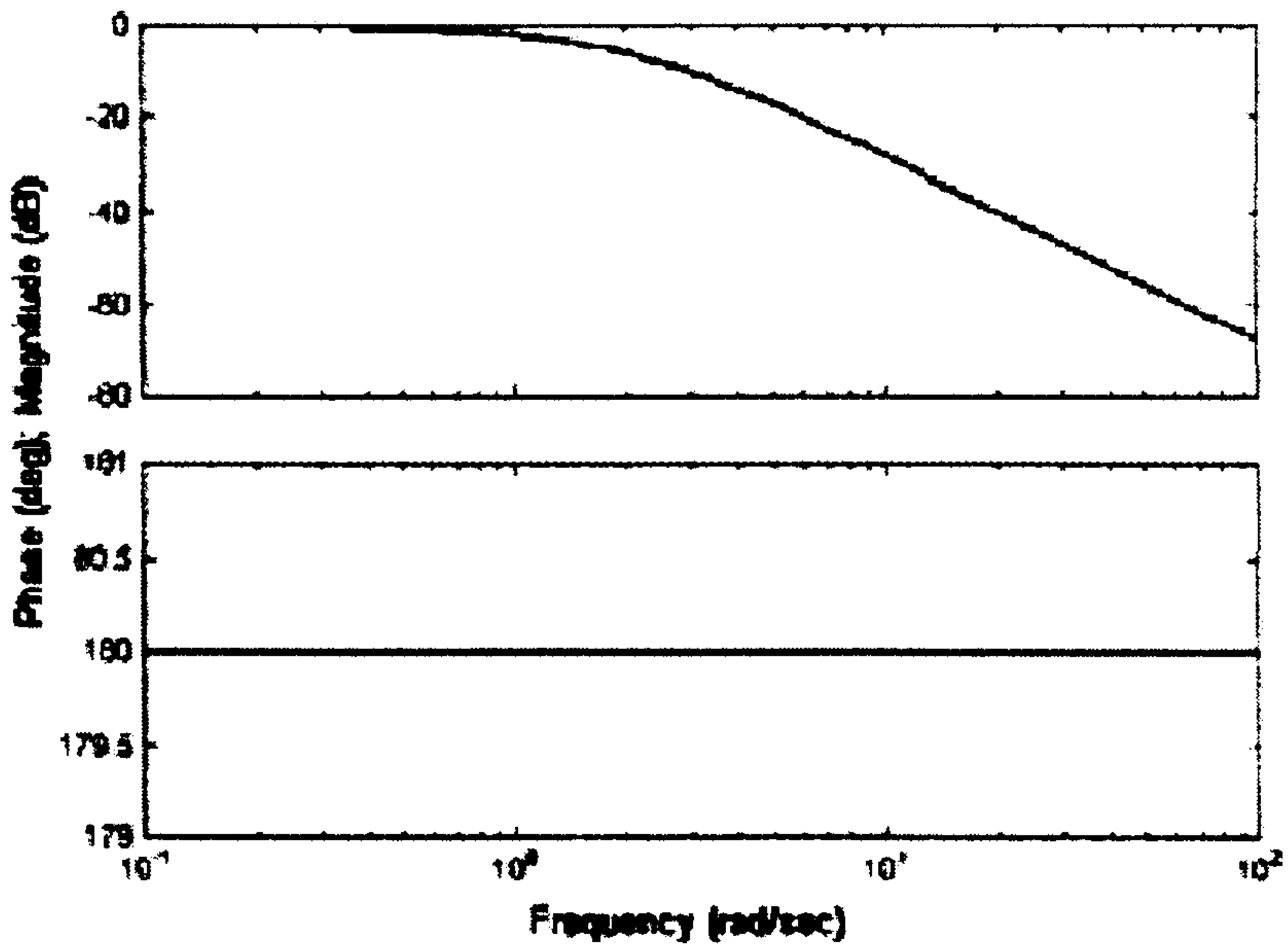


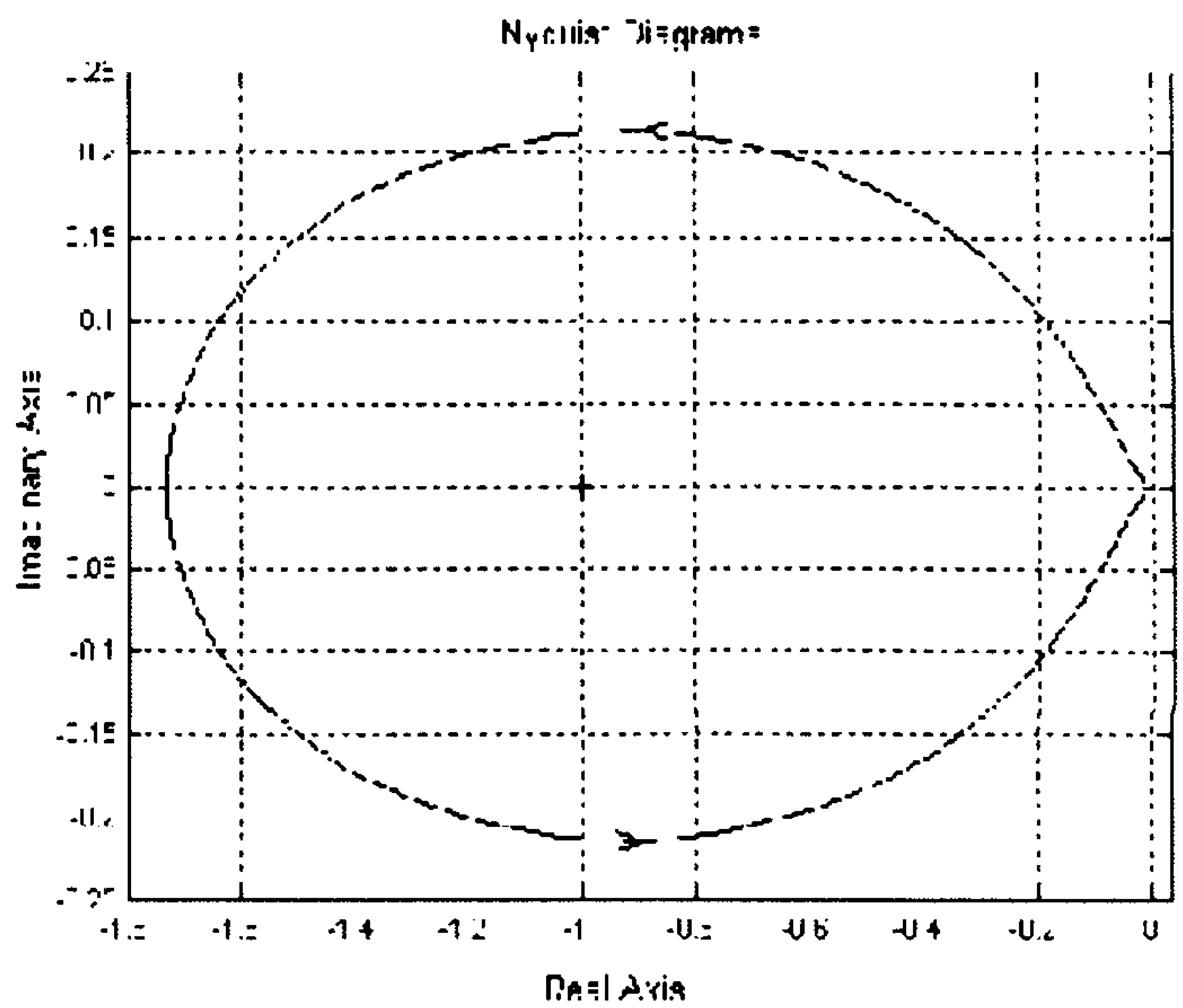
Figure 2(b)



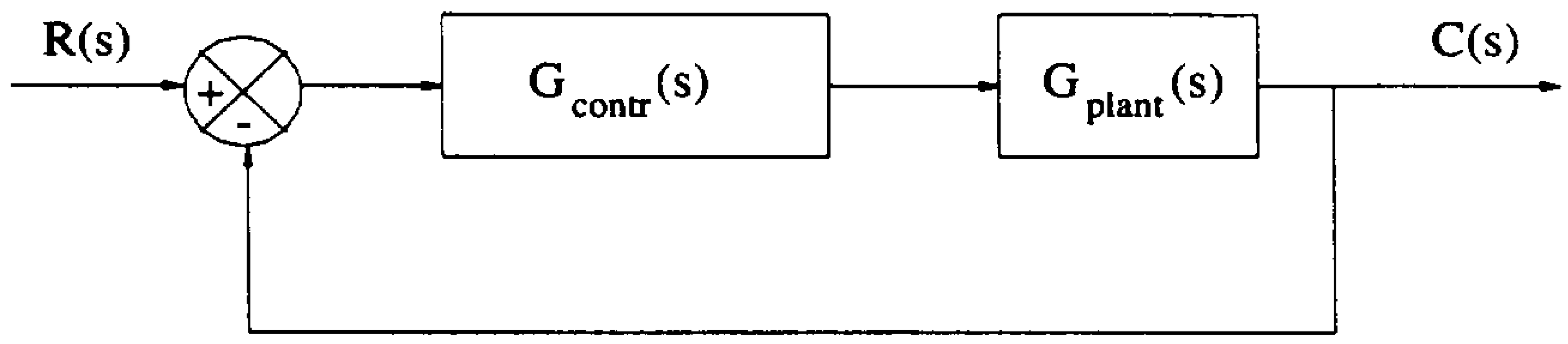
(a) Given that *r(t)* is a step input of 2 units, design a PD controller such that the closed-loop system is critically damped and has a natural frequency of 10 radians per second. Sketch the corresponding step response of y(t) neatly.

(b) We then replace H(s) by a low-cost sensor that can be characterized as a low-pass filter. Figure 2(c) is a Nyquist (or polar) plot for the resulting system. Determine the stability of the position control system.

Figure 2(c)



Problem 3:
Consider the Block diagram below:



(a) It is

$$G_{\text{plant}}(s) = \frac{s+\frac{1}{2}}{s^3+3s^2}$$

Assume that simple proportional control is used, i.e. $G_{\text{contr}}(s) = K$.

Use Routh's stability criterion to determine for which values of K (if any) the real parts of *all* closed-loop poles are to the left of (-0.5), i.e. Re(CL poles) < -0.5.

(b) Sketch the root locus for the system described in (a), i.e. with $G_{\text{contr}}(s) = K$.

Include arrow heads to indicate the direction of increasing K.

(c) Design a new controller, $G_{\text{contr}}(s)$, using your root locus plot by adding the **minimal** number of OL poles and/or OL zeros, such that the CL system is asymptotically stable and does never exhibit oscillations.

Explain, using a root locus plot, how to choose the controller and choose all constants explicitly for $G_{\text{contr}}(s)$. (If it is not possible to design such a controller, explain why it is impossible.)

(d) In this last part consider the following controller:
$G_{\text{contr}}(s) = K_p + K_d s$, where $K_p = -2, K_d = 2$.

(i) What is the type of the system?

(ii) What is the steady-state error of the CL system for a unit step?