GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Quarter 1997

System Dynamics & Controls
EXAM AREA

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Assigned Number (DO NOT SIGN YOUR NAME)

- Please sign your name on the back of this page—

George W. Woodruff School of Mechanical Engineering

Spring 1997 Doctoral Qualifying Examination

INSTRUCTIONS

There are 4 questions attached, please solve all questions as completely as possible. State all assumptions, and make sure that you clearly indicate the thought processes that you employed to arrive at your answer.

CONTROL DESIGN PROBLEM

Determine the controller transfer function, K(s) such that the closed-loop system shown in Figure 1 meets the following specifications:

1. No Overshoot in Y to a step in R.
2. 1% settling time of 2 seconds.
3. Zero Steady State Error to a Step in R.

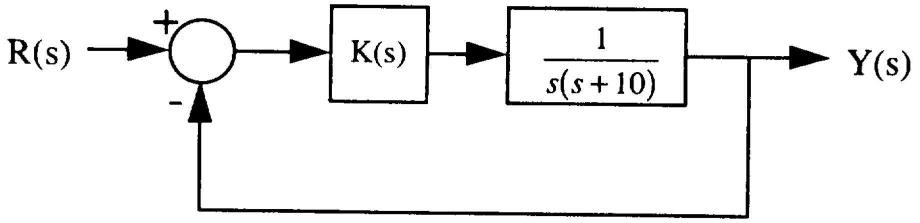

Figure 1. System for the Control Design Problem.

You should note that if your solution has a zero in it, it may affect your intuition as to meeting the requirements. Make sure you discuss this.

The following Pade' function is often used to approximate transport lag in control system design:

$$G(s) = \frac{(Ts)^2 + -6(Ts) + 12}{(Ts)^2 + 6(Ts) + 12}$$

Sketch the polar plot of the above Pade' function and show that for the frequency range $0 < \omega T < 2\sqrt{3}$, the Pade' function gives a good approximation to the transport lag e^{-Ts}.

Qualifying Exam: Root Locus

The position control system you must design can be modeled at various levels of detail. In the simplest model the plant can be represented as a pure inertia. In more refined models the actuator dynamics and structural elasticity of the moving parts are also to be considered.

a) Consider the plant to be a pure inertia J. For the closed loop system the measured position is subtracted from the desired position signal and fed into a P.D. controller with transfer function

$G(s) = K\,(K_P + K_D s)$

Choose K_P and K_D so that when $K = 1$ the closed loop system will have a settling time of 0.1 sec. Sketch the root locus as K varies.

b) Consider selecting the gains so that the damping ratio is 0.707. Sketch the root locus as this damping ratio is maintained but the natural frequency of the closed loop system is varied.

c) We find that when compliance is considered the plant transfer function becomes

$$G_p(s) = (s^2 + \omega_z^2) / [s^2 (s^2 + \omega_p^2)]$$

What is the effect on the root locus for part a) if

1. $\omega_z > \omega_p$

2. $\omega_p > \omega_z$

Explain the behavior expected based on a sketch of the modified root locus.

d) If the actuator is a first order lag with a fairly "fast" time response, what is the effect on the root locus in a)?

Modeling Question:

The engineers at Dupont have to design the braking system for one of their ultracentrifuges. In this system, a rotor of rotational inertia, J_R, must be decelerated from an initial rotational speed of ω_0. In order to accomplish this, they shunt the electric motor across a series of power resistors, R_R, and rely on the back EMF to slow the rotor down. Some information you may need in formulating this problem: back EMF constant for the motor is K_b, motor-torque constant is K, inertia of the motor armature is J_a, armature resistance is R_a, and you can ignore the inductance of the armature and any bearing friction.

Part A
Draw a schematic diagram for this system including both the mechanical and electrical components and write out the governing equations for the system.

Part B
Develop a differential equation for the back-EMF developed in the motor and solve this for the back-EMF as a function of time.

Part C
Is it possible to obtain a uniform (constant) deceleration from this system design? Why or why not? Please support your answer with evidence from the governing equations that you developed in Part A.

Part D
Solve for the value of R_R that will drop the rotor speed to approximately 1% of ω_0 in 3 minutes after the brake is engaged (assume t is measured in seconds). What is the instantaneous power dissipation in the power resistors, R_R? Recall that P=EI.