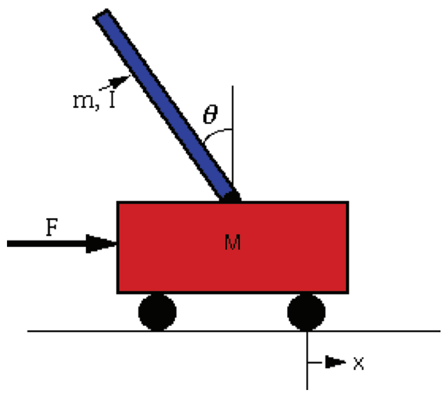
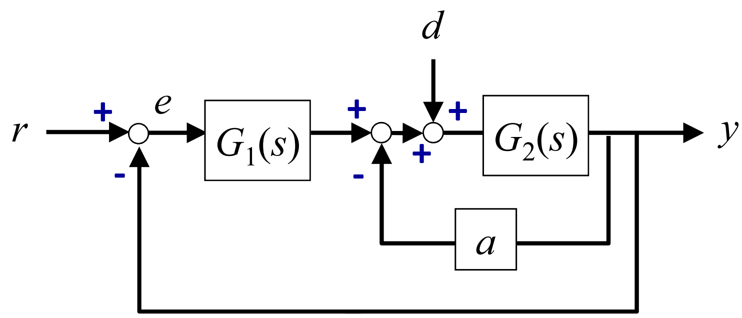
Work 3 out of 4 questions only.

1. An inverted pendulum is attached to a moving cart. The pendulum is a uniform rod. The cart is moving on a smooth surface. The angular position of the pendulum is $\theta$ and the cart is subject to a horizontal force $F$.

Obtain the transfer function of the system. The input may be $F$ or any quantity directed related to $F$. The output may be $\theta$ or any quantity directly related to $\theta$. Apply linearization method when necessary. Define any other quantities when necessary.



2. Consider the following closed system




where $G_1(s)=\dfrac{g}{s+a}$ and $G_2(s)=\dfrac{1}{s+1}$. $a$ and $g$ are constants.

(1) Obtain transfer functions from $r$ to $e$ ($G_{re}(s)$) AND $d$ to $y$ ($G_{dy}(s)$) .

(2) Find the condition(s) between parameters $a$ and $g$ such that the closed system is stable.

(3) Assume that the feedback system is stable and $r$=0. Determine the necessary and sufficient condition for $\lim_{t\to\infty} e(t)=0$ when a step-input is given for $d$.

(4) Assume that the feedback system is stable. Determine the condition(s) between $a$ and/or $g$ such that the closed system is critically damped.

3. The figure shows a block diagram of a dynamic system where *a* is a variable parameter.

a) Sketch a graph of the closed-loop poles for $a \geq 0$.

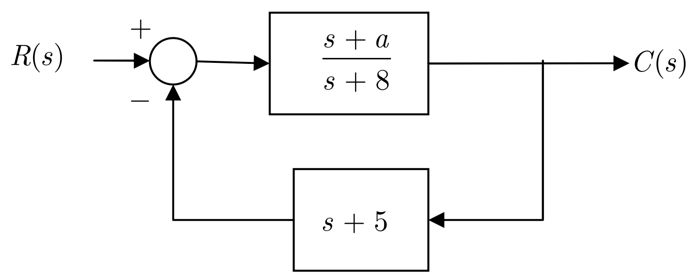
b) Determine *a* so the response will be critically damped.

c) For the result in b) is it possible to have overshoot in response to a unit step input? Explain.

d) What value of *a* would you expect to result in the fastest settling time? Explain.

e) On the graph show the location of the closed-loop poles that you would expect to result in the greatest rate of oscillation? Explain.

f) Sketch a representative time response to a unit step for very large values of *a*. Explain.



4.

Figure 1 shows a cherrypicker that is used to raise workers to high heights. The machine is essentially a robotic arm that is driven by a person riding in a basket at the end of the arm. If the basket oscillates, then it is difficult for the person to work safely and efficiently. On rare occasions, the whole machine tips over and kills the person.

**Figure 1: Cherrypicker.**

1) Create a simple model that can predict the basket oscillation when the machine is subject to wind disturbances. The model should contain terms that account for the changing configuration of the arm and the changing mass of the endpoint basket that occurs with different operators, tools, and supplies.

2) Sketch the payload time response when wind creates i) a low-frequency sinusoidal excitation, and i) a high-frequency sinusoidal excitation. (Low-frequency is below the important modes, high-frequency is above them.)

3) Sketch the Bode plot corresponding to your model.

4) Develop a control system to decrease the basket oscillation.

5) Sketch the Bode plot corresponding to the system with your controller.