ME Ph.D. Qualifying Examination — System Dynamics and Controls, Fall 2008

Choose **3** of the following **4** questions to answer.

Problem 1
Consider a dynamic system whose transfer function is given by

$$G(s) = \frac{1}{(s+1)^3}.$$

(1) Determine the gain and phase margins.

(2) Suppose the input to the system is $2\sin 5t$. Find the steady-state response.

Problem 2

Consider the unity-feedback PD controlled system with an open-loop transfer function,

$$G(s) = \frac{K(1+Ts)}{s(s+1)(s+2)}.$$

(a) Use a root locus plot to illustrate the effect of both $T(\geq 0)$ and $K(\geq 0)$ on the dominant closed-loop poles. Specifically, plot the root locus for $T=0$, and use this plot as a guide to sketch the loci for $K=3$, 6, and 20.

(b) Use $K=6$ for this part. Show, using the definition of phase margin, that T has an effect on improving the closed-loop system stability. Find T such that the phase margin is 45°.

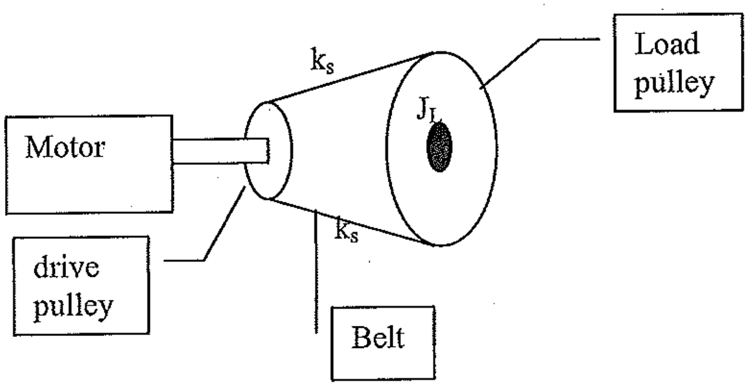
Problem 3

A permanent magnet d.c. motor is driven by an applied voltage v_a to produce current i_a which results in torque $T = K_T i_a$. (K_t is a constant). The applied voltage is effectively reduced by back emf $e = K_B \omega$ when the motor shaft is turning. The armature circuit is comprised of resistance R_a in series with an inductance L_a in addition to the motor back emf source.

(a) Draw the block diagram to represent the system above in which the inputs are v_a and ω and the output is torque T. Show all transfer functions in terms of the nomenclature above.

(b) The motor shaft is now attached to a drive pulley which when combined with the motor armature has radius r_m and rotational inertia J_m. On the pulley is mounted a belt with compliance as shown in the drawing below. Model the system and determine ω_L, the rotational speed of the load pulley on the other end of the belt. Model the system as if each length of the belt has a spring constant k_s and the belt is always in tension on both sides. The radius of the load pulley is r_L and it has rotational inertia J_L.

Determine the transfer functions and block diagram that will predict ω_L given the input v_a.



(c) Under high torque, it may be possible for tension on one side of the belt to go to zero. What is the consequence of this on the system model, in particular consider the system poles assuming the condition persists and a linear model is still valid.

(d) If the parameters of the load (belt and pulley) are unknown, propose experiments that would enable you to determine them to a good approximation. In addition to the parameters dictated by the model above, what additional parameters and corresponding behaviors might be of interest?

Problem 4

South Korea is developing a permanent storage center for their nuclear waste. The figure below shows a schematic representation of their design for a below-ground storage silo. An overhead crane will pick up a waste container from a truck and transfer it over to the silo. It will then lower the container down into the lowest available storage slot. There will be 5 different sizes of the storage containers, so the stacking pattern will be somewhat irregular. The designers want to have a positioning accuracy of 1 cm for the storage containers. They also want the process to be completely automated from removing the container from the truck all the way until final positioning in the silo.

1) Describe a measurement system that could locate the position of the waste container on the truck to within 5 cm in horizontal directions. That is, what sensors could accomplish this task and how would those measurements be used?
2) When the crane attempts to pick up the container, there will be some sway in the crane. How does the dynamics of the sway change after a heavy (16 ton) container is grabbed by the crane?
3) What kind of control system would you use to move the container from the truck to the top of the silo? Sketch a block diagram of your control system.
4) When the container is lowered far down into the silo, how do the dynamics change, as compared to when it is at the top? Address effects in three dimensions of translation and rotation.
5) How would use change your control system operation when the container is at the bottom of the silo?
6) How can you achieve 1 cm positioning accuracy at the bottom of the silo? That is what control elements would be needed to achieve this challenging performance specification?