GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - FALL Semester 2001

System Dynamics & Controls
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

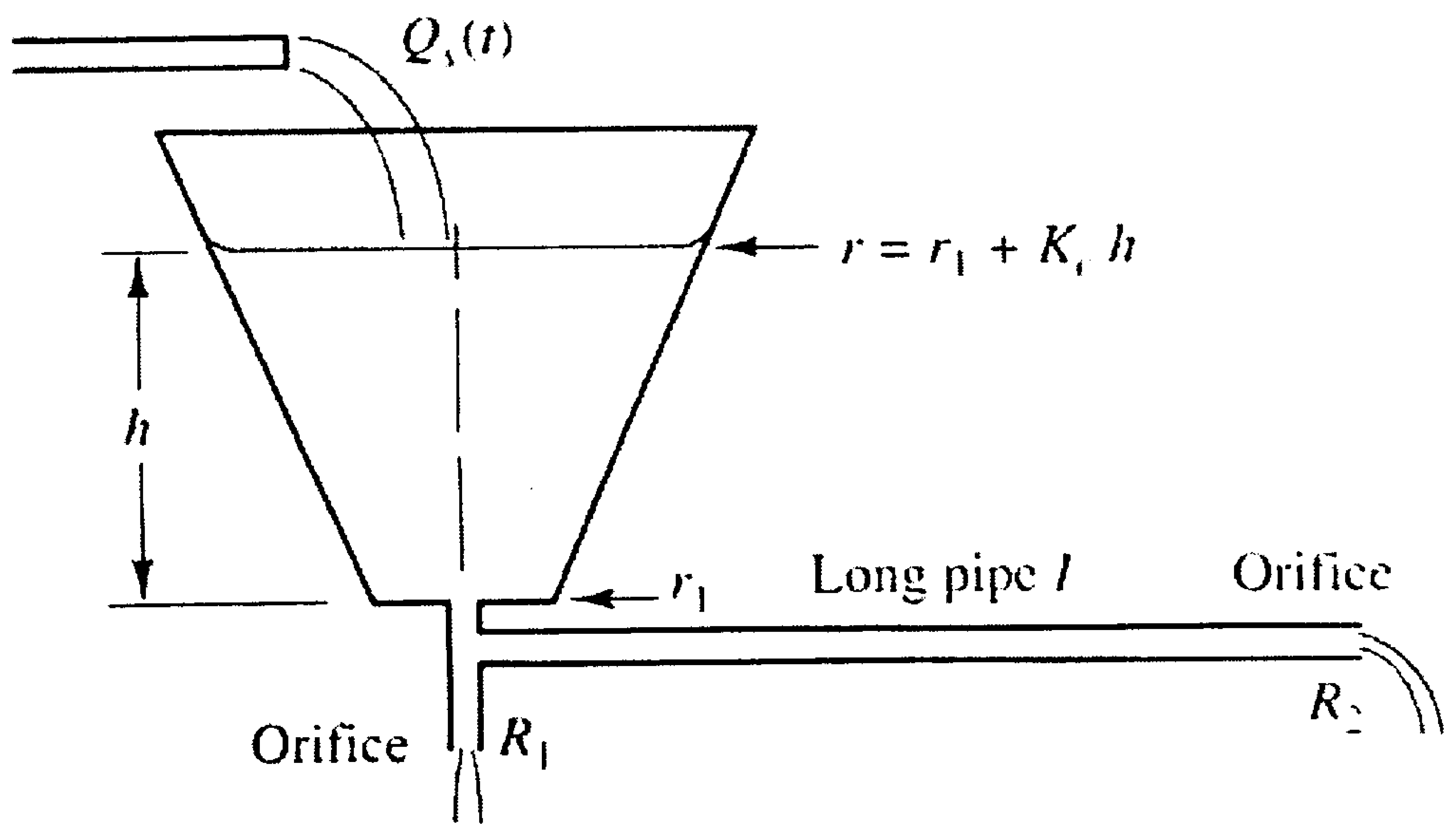
Problem 1

The fluid distribution system shown below consists of a flow source $Q_s(t)$ which feeds a storage tank with nonvertical walls. The output from the tank is distributed into a fluid network consisting of a short pipe discharging through an orifice and a long pipe discharging through another orifice. The fluid flow through an orifice obeys a quadratic relationship:

$$Q = C_0\sqrt{|\Delta P|}\,\mathrm{sgn}(\Delta P)$$

where Q is the flow through the orifice, ΔP is the pressure drop across the orifice, and C_0 is an orifice coefficient that is dependent on the geometry of the orifice. The signum function, sgn (), is used to indicate that the flow changes sign when the sign of ΔP changes. In the figure below, the parameter, I, denotes the fluid inertance of the long fluid line. Since we are interested to observe the pressure $P_c(t)$ at the base of the tank, and the flow $Q_l(t)$ through the long pipe (eecall that the inertance I represents fluid inertial effect= $P_I/\dot{Q}_I$, where P_I is the pressure difference across the pipe). Assuming that the pipe resistance is small compared to the orifice resistance, derive

(a) a set of state equations, and
(b) the transfer functions relating P_c and Q_l to the input source Q_s.



Problem 2

For the systems shown in Figure 1, the relationship between force and position is given by

$$\frac{X(s)}{F(s)} = \frac{1/m}{s\left(s+b/m\right)} \qquad (1)$$

Part A

Using root locus techniques, design a controller, K(s), to achieve the following specifications:

1. Zero (0) steady state error to a step input in force, F.
2. A damping ratio of 0.5.
3. A natural frequency of 2 rad/s.

Your controller should be the lowest order controller possible. Make sure to draw the root locus as a function of the forward loop gain for your controller. Please specify all controller parameters in terms of the parameters m and b.

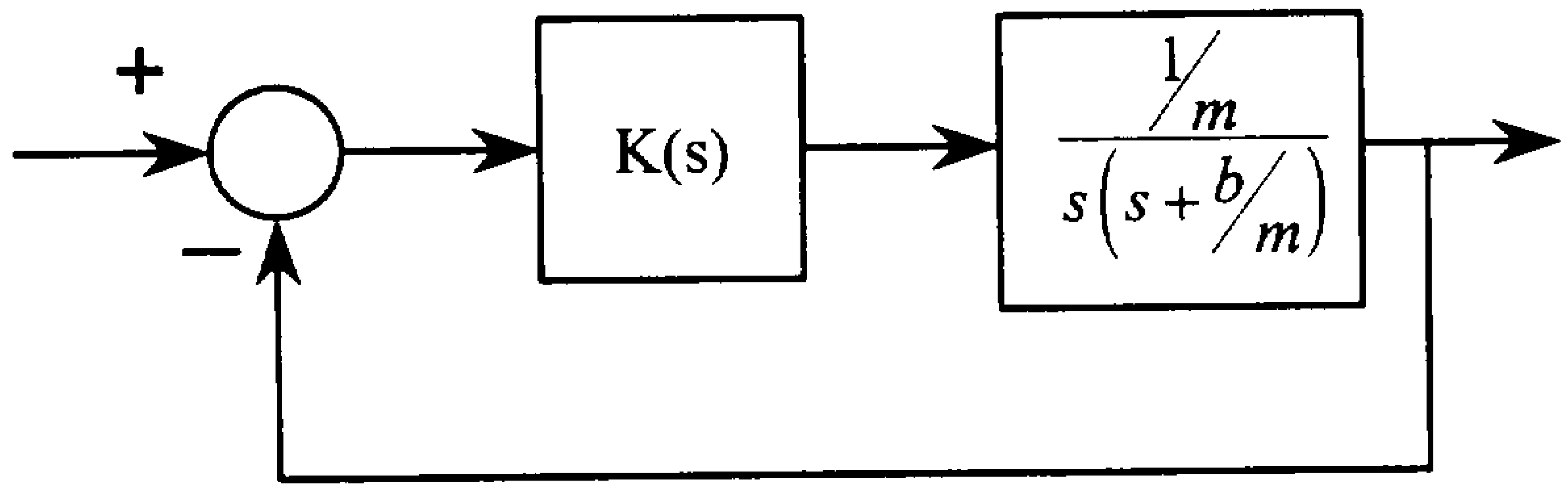


Figure 1: A Simple Model for the Problem.

Your controller should be the lowest order controller possible.

Part B

In less than one half of a page, discuss how your answer would change if specification #2 in Part A (the damping ratio of 0.5) were changed to "A percent overshoot of XX." Please note that

$$M_p = blah \qquad (2)$$

Part C

Now assume that your system is a bit more complex as is your controller design. You are given the unity gain feedback configuration shown in Figure 2. The open-loop pole / zero plot for the system described in Figure 2 is presented in Figure 3.

Please sketch the root locus of the system shown in Figure 2 as a function of the loop gain, K.

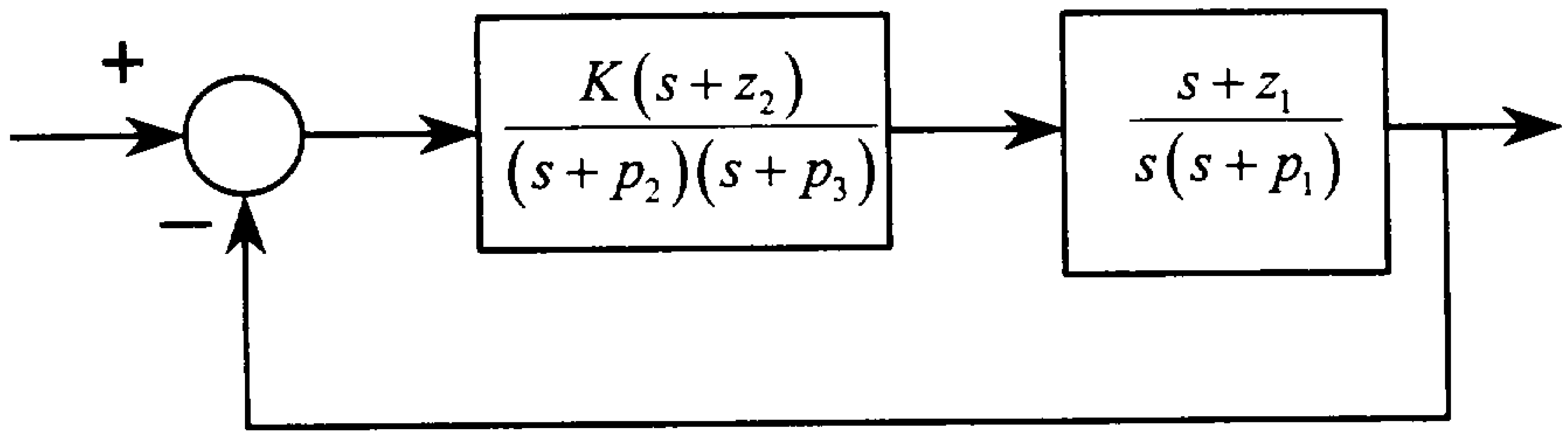


Figure 2: A More Complex Model for the Problem.

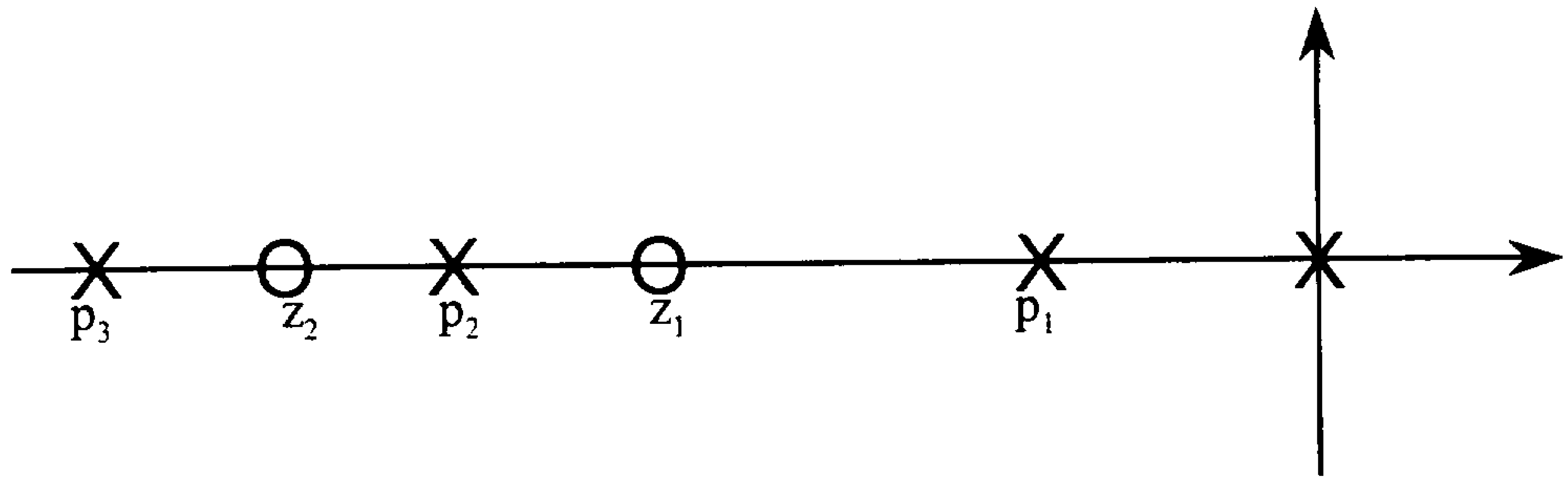


Figure 3: Open-Loop Pole / Zero Plot for the More Complex Model.

Part C

Clearly some poles in your root locus plot for Part C migrate to a magnitude of ∞ as K→∞. Let us call these poles the infinite poles. Please provide a mathematical expression for the rate at which the infinite poles migrate to a magnitude of ∞ as a function of K.

Part D

Clearly some poles in your root locus plot for Part C migrate to the zeros as K→∞. Let us call these poles the finite poles. Please provide a mathematical expression for the rate at which the finite poles migrate to the zeros as a function of K.

Problem 3

A system G is placed inside a feedback loop with a controller with transfer function G_c. The Bode response of G is shown in the figure below.

(a)If G_c is a constant gain, find the value K_1 of that gain such that for all positive gains less than K_1 the close loop system is stable. Justify your answer in terms of the frequency response alone.

(b) Find an approximate transfer function for G based on the frequency response as given in the Bode plot.

(c) What will be the steady state response of the system with a gain just below the limiting value found in (a) for an input R(s) that is

A step input $1/s$.

A ramp input $1/s^2$.

(d) Consider gains 10% larger and 10 times larger than the limiting gains. What is the steady state error for these cases for the ramp input? Explain your answer fully.