GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Quarter 1995

System Dynamics + Controls
EXAM AREA

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Assigned Number **(DO NOT SIGN YOUR NAME)**

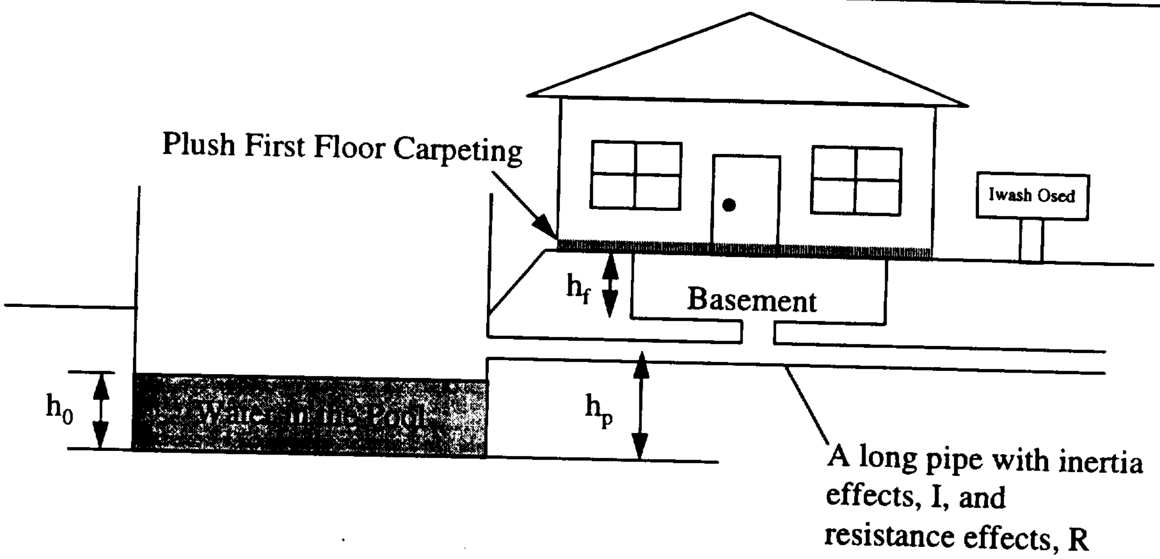
-- Please sign your name on the back of this page --

Dynamics and Control Qualifier

There are four problems attached, so you have an average of 30 minutes to work on each problem. We are trying to determine your understanding rather than your memory of obscure formulas or your ability to calculate a particular result. Therefore, it is important that you make clear the method you are using and the reasoning process. Try to show an orderly approach. Please, make the text easy to read. Cross out errors rather than try to erase. If a problem seems to require a difficult set of calculations, you are probably attempting the wrong approach or attempting to generate too precise a result. E.g., plots and drawings don't need to have much precision. Some problems don't require numbers at all but rather a very short discussion.

Don't let a particular problem or subsections of a problem take too much time. Go on to something else and come back.

Dr. Iwash Osed has built a house and has done a rather silly thing. He has connected the drain in his basement to the overflow drain in his pool which is located down the hill from his house. Now Hurricane Buzz is preparing to drop quite a bit of rain on Dr. Osed's house and pool. Being a theoretically precise hurricane Buzz is dropping rain at a constant rate (e.g., F mm/hr where F is constant). Dr. Osed is not worried about his basement flooding; however, he really does not want the carpeting to be ruined on his first level. Your mission, which you must accept, it is to determine if Dr. Iwash Osed's first floor will flood. Of course, this will depend on the *constant* rate of rain fall that Buzz is generating, F (mm/s) and how long the rain is falling, T (sec). Please use only the parameters in the figure, assume that water has a density of ρ, and that light travels at 186,282.397 miles/sec. Assume that the pool is initially filled to a depth of h_0. Assume that the pool has a cross sectional area of A_p and the basement has a cross sectional area of A_b. (Note that Dr. Osed's first floor is slightly below the top of the top of the pool.)



Iwash Osed's House.

Consider the closed-loop system shown in the figure.

Part a

Suppose that

$$G_c(s) = 1, \qquad G_p(s) = \frac{(s+6)}{s(s-4)}.$$

What is the response $c(t)$ when $r(t)$ is a unit-step input?

Part b

Explain if you can justify the answer when $t = \infty$ by using the final-value theorem.

Part c

Suppose that

$$G_c(s) = K_P + K_D s, \qquad G_p(s) = \frac{(s+6)}{s(s-4)}.$$

That is, we decide to use the PD control. We want to determine the feasible range

$$a < K_P < b, \qquad c < K_D < d$$

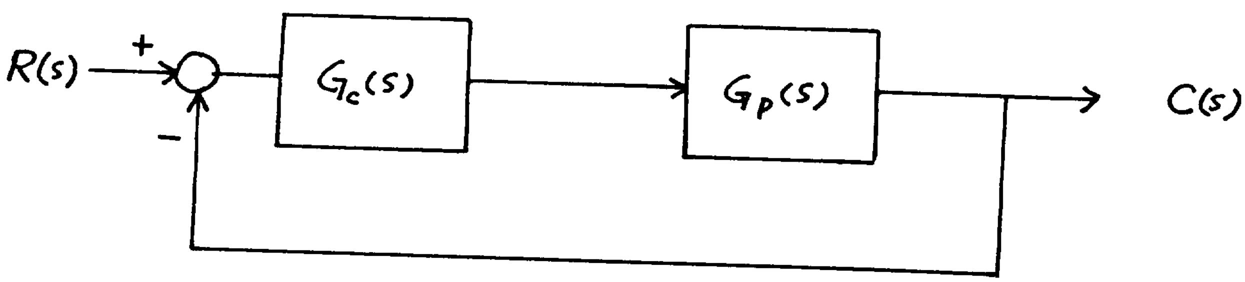
such that the closed-loop system is stable. What are then (the least conservative estimate of) a, b, c, and d?

Part d

Suppose that

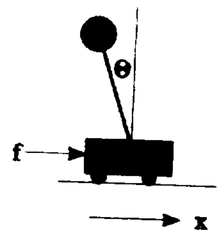
$$G_c(s) = K, \qquad G_p(s) = \frac{(s+2)}{s(s+1)}.$$

Estimate the suitable gain K so that the overshoot of the step response is approximately 15%.



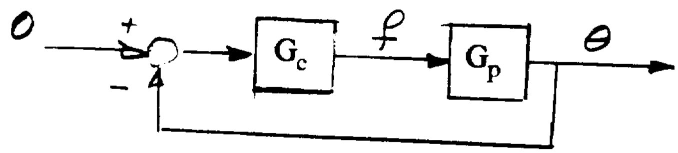
An inverted pendulum has a transfer function relating the force on the base, f, to the angle of the pendulum, Θ, of

$$\Theta = \frac{1}{s^2-1} f$$



Part a

Joe Simple says that all we need to do to make a stable system is to cancel the unstable pole and add a gain as shown here.



where $G_c = K(s - 1)$ and $G_p = \frac{1}{s^2-1}$.

Will this work? Explain.

Part b

Mo Complex suggests that we could cancel the stable pole put in a little gain. In this case $G_c = K(s + 1)$. Will this work? Explain.

Part c

Now lets consider a G_c that is described only by its frequency response. What fundamental requirement must be met by the product $G_c G_p$ for the closed loop system to be stable?

Part d

The frequency response of G_p is given by a simple graph in a polar (or Nyquist) plot. Sketch that plot.

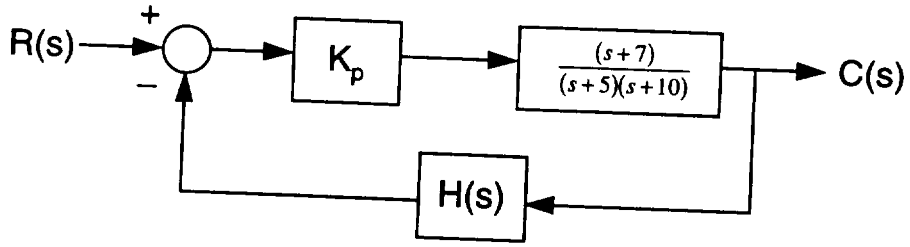
Part e

Now show how G_c must modify this plot to give a stable feedback system. Use one of the two suggestions given earlier.

Part f

In this problem, assume you are able to stabilize the feedback system so that Θ is eventually brought to zero. If x is initially zero, but Θ isn't, what do you think happens to x(t). No numerical answer is expected here, just a general description of the nature of the function x(t).

A system for controlling the roll angle for a submarine has the block diagram relationship



Part a

If H(s) is given by the all pass filter

$$H(s)=\frac{(s-2)}{(s+2)}$$

Sketch the root locus for the system as a function of the forward loop gain, K_P.

Part b

Now let us say that H(s) is

$$H(s)=\frac{(2-s)}{(s+2)}$$

Sketch the root locus for the system as a function of the forward loop gain, K_P.

Part c

Discuss the differences between your results for parts a and b.

Parts d & e

Finally, we will replace the proportional compensator, K_P with a PID compensator of the following form

$$K(s)=K_D s+K_P+\frac{K_I}{s}$$

Part d

Let $K_P=10$ and $K_I=8$, please draw the root locus plot, as a function of the gain, K_D.

Part e

Let $K_P=10$ and $K_D=4$, please draw the root locus plot, as a function of the gain, K_I.