Problem 1

Two well insulated containers of the same volume are linked through a thin pipe with a valve, as shown in the following figure. Initially, container A is filled with n moles of an ideal gas at pressure p_1 and temperature T_1. The specific heat ratio of the gas is $\gamma = c_p / c_v = 1.4$, which may be treated constant in the temperature range considered. Container B is empty initially.

The valve in the pipe is opened and pressure equilibrium is quickly reached and the valve is immediately closed. Assume that the containers and pipes are well insulated, and the volume of the pipe can be neglected. The objective is to develop a set of equations that can be used to find the properties of the final states: the amount of gas that is transferred to container B, n_B in moles; the final pressure, p_2, in both containers; and the final temperatures of the gas in containers A and B, i.e., T_{A2} and T_{B2}, respectively.

To better understand the process, you are asked to fill the blanks first. Right after the valve is opened, a small amount of gas enters container B. The temperature of the gas in container B will be ________________ (higher than, lower than, the same as) T_1. As more gas flows into container B, the temperature of container A will ____________ (increase, decrease, remain the same), whereas the temperature of container B will ____________ (increase, decrease, remain the same).

Derive each of the following equations:

$$\frac{T_{A2}}{T_1} = \left(1 - \frac{n_B}{n}\right)^{\gamma - 1} \quad \text{(A)}$$

$$\frac{T_{B2}}{T_{A2}} = \frac{n - n_B}{n_B} \quad \text{(B)}$$

$$\frac{1}{T_{A2}} + \frac{1}{T_{B2}} = \frac{2}{T_1} \quad \text{(C)}$$

$$p_2 = \frac{1}{2} p_1 \quad \text{(D)}$$

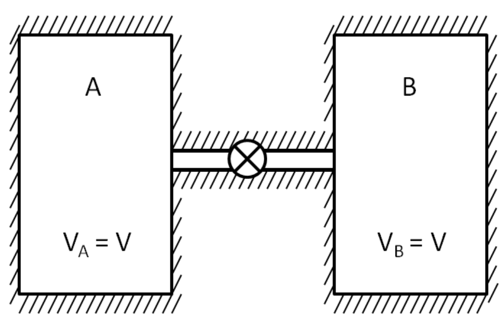
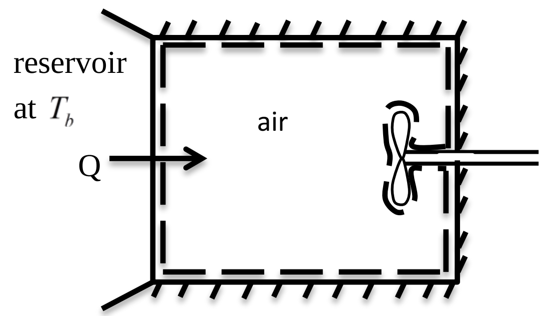


Figure for Problem 1.

Please make your derivations and assumptions clearly since partial credits will be given.

Problem 2



Consider a system consisting of air in a rigid container fitted with a paddle wheel, which may be placed in contact with a thermal reservoir, as needed. By heating (from the reservoir) and/or stirring (with the paddle wheel) the temperature of the air can be increased from T_1 to T_2 in alternative ways. We want to explore how the temperature increase can be achieved with maximum entropy production and how it can be achieved with minimum entropy production. Assume that whenever heat transfer occurs, the associated boundary temperature, T_b, is the same as the reservoir temperature, and $T_1 < T_b < T_2$. The air can be treated as an ideal gas with constant specific heats.

For maximum entropy production, find

i) the heat transferred, Q

ii) the work done by the paddle wheel, W_P

iii) the entropy production, σ

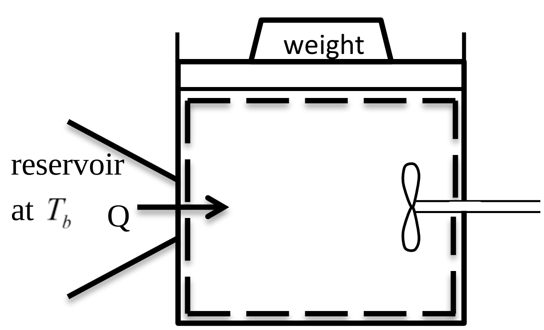
For minimum entropy production, find

iv) the heat transferred, Q

v) the work done by the paddle wheel, W_P

vi) the entropy production, σ

Now consider the system below, in which the air is contained in a vertical cylinder.



vii) With the same assumptions as the previous case, how would your answers to parts *i - vi* change?

Problem 3

Helium, which has molar mass = 4.00 kg/kmole and can be taken to be an ideal gas with constant specific heat ratio equal to 5/3, is the working fluid in a closed steady-state gas power cycle. The compressor (efficiency = .85) and the turbine (efficiency = .90) are isothermal (constant and uniform temperature) machines. These efficiencies are defined by comparison with the corresponding reversible isothermal processes. The compressor and the compression process are at uniform T_L and the turbine and the expansion process are at uniform T_H. The cycle includes a recuperator heat recovery exchanger and a high temperature heat input (HHEX) exchanger and a low temperature (LHEX) heat rejection exchanger. As implied in the accompanying figure, the recuperator heats gas at T_L leaving the compressor through some fraction f_{HX} (from 0 to 1.0) of the temperature difference $T_H - T_L$. The gas is then heated to T_H in the HHEX with external heat. Ignore pressure drops outside the machines. Assume the high pressure in the cycle is 2000 kPa and the low pressure is 200 kPa. Assume the high temperature T_H is 800 C and the low temperature T_L is 25 C and that the recuperator heats the gas through 80% of the temperature difference $T_H - T_L$.

Complete the following table:

station	temperature (C)	pressure (kPa)		
1				
2				
3				
4				
5				
6				

Calculate the following

(1) Turbine work per unit mass = ____________ kJ/kg
(2) Compressor input work per unit mass = ____________ kJ/kg
(3) Recuperator heat exchanged per unit mass = ____________ kJ/kg
(4) Heat input to HHEX exchanger per unit mass = ____________ kJ/kg
(5) Cycle thermal efficiency (net work / heat input) = ____________

Assume the reference state for zero entropy is 25 C and 100 kPa and calculate the following

(1) Specific entropy at compressor inlet (station 1) = ____________ units = ________
(2) Specific entropy at compressor outlet (station 2) = ____________ units = ________

Assume the gas flow rate is 1.0 kg/s and do the following

(1) Compute the compressor power input = ____________ units = ________
(2) Find the rate of entropy generation in the compressor = ____________ units = ________
(3) The effectiveness of the recuperator heat exchanger = ____________ units = ________