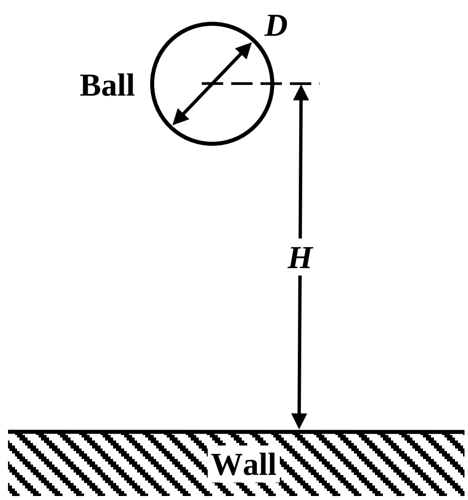
1) An elastic spherical ball of diameter D is dropped from a height H with an initial velocity of 0 (zero) and hits a rigid horizontal wall. The area of contact between the ball and the wall during impact is A, and the impact contact time (*i.e.*, the time during which the ball touches the wall) is τ. The density of the ball is ρ, the elastic Young's modulus of the ball is E (with units in basic dimensions of $ML^{-1}T^{-2}$), and the Poisson's ratio of the ball is γ. The acceleration due to gravity is g.

a) You wish to repeat the experiment using a smaller ball made of the same material as the original ball, but with a diameter of $0.5D$. Based on dimensional analysis, from what height h will you need to drop the ball to have the same relative deformation of the ball during impact with the wall?

b) How does the impact contact time Δt in the repeat experiment described in part (**a**) compare with that for the original experiment, τ?



2) Consider a cube of ice of dimension L made of fresh water floating in a body of water.

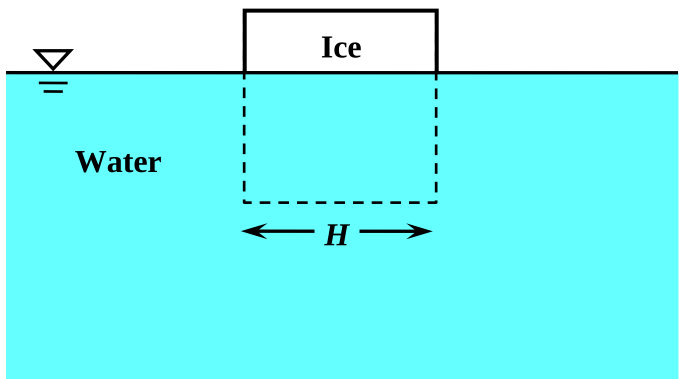
a) The ice is floating in a freshwater lake. If the density of fresh water is ρ, and the depth of the submerged part of the cube of ice is CH (where $C < 1$), what is the density of the ice ρ_i?

b) Will the level of water rise once the ice melts?

c) Now, consider the same cube of ice floating in the ocean. If the depth of the submerged part of the cube of ice is DH (where $D < C$), what is the density of the salt water ρ_s?

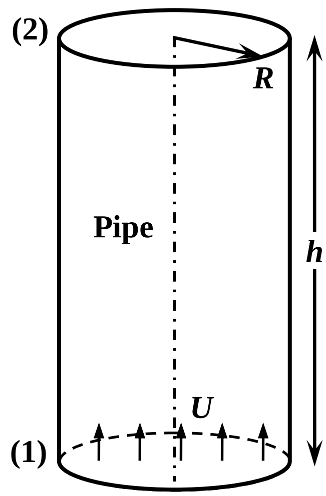
d) Will the level of water rise once the ice melts?

e) Based on your result, how will global warming will affect the polar ice caps?



3) Water of density ρ flows vertically upward in a stationary vertical section of a pipe with inner radius R and of axial length h. The velocity profile at the inlet, section (**1**) is that for uniform flow with a speed U; the velocity profile at the exit, section (**2**), is parabolic, or that for fully-developed laminar pipe flow. The known parameters in this problem are ρ, R, h, U, the pressure difference $\Delta p \equiv p_1 - p_2$ from section (**1**) to section (**2**), and the weight of the pipe section W.

Find the axial force $\vec{\mathbf{R}}_z$ exerted by the pipe on the water in terms of the known parameters.

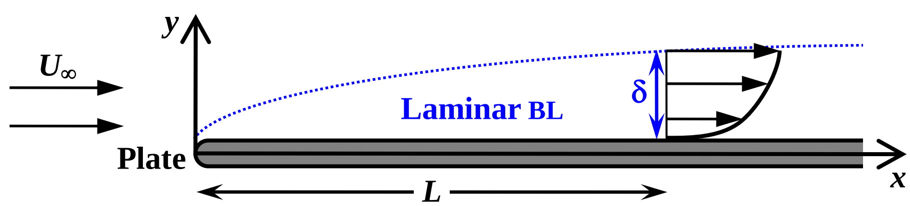


4) The term ***boundary layer*** refers to a thin layer of viscous fluid adjacent to a solid wall, where the velocity parallel to the wall "smoothly" increases from zero to the freestream value U_∞. Consider a laminar boundary layer (BL) over the upper surface of a plate governed by the (dimensional) equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

where u and v are the velocity components along the x- and y-directions, p is the pressure, ρ is the fluid density and ν is the fluid kinematic viscosity.



a) A common way to define the thickness (*i.e.*, y-dimension) of the boundary layer is to use the distance normal to the wall where the velocity recovers to 99% of the freestream value, so that at $y = \delta$, $u/U_\infty = 0.99$. This definition of the BL thickness is, however, arbitrary (for example, why 99%? why not 90%?). Please provide an alternative definition of the thickness of the BL that is based on a basic principle of fluid mechanics, and identify that basic principle.

b) In general, the BL will ***grow***: *i.e.*, the thickness of the BL will increase as x increases. Please provide a brief physical explanation of why the BL grows. If the pressure gradient $dp/dx = 0$, the BL thickness will be proportional to x^N. Use a scaling analysis to show that $N = 1/2$.

c) Although the flow is primarily along the streamwise direction, with $u \gg v$, the normal velocity component v cannot be neglected. Show that setting $v = 0$ in the governing equations for a laminar BL will preclude, or make impossible, the growth of the boundary layer.

d) An ***adverse pressure gradient*** is, by definition, a gradient where $dp/dx > 0$. Using the governing equations, briefly discuss how an adverse pressure gradient will affect the shape of the velocity profile $u(y)$ near the surface of the wall.