Two configurations of a thin-walled, two-dimensional hollow partition are considered for holding liquid of density ρ at level H within a reservoir, as shown schematically below. The partition can rotate along a hinge at its bottom edge which also seals the reservoir. Due to the presence of the hinge, there is a narrow gap of height d between the bottom of the partition and the bottom of the reservoir (d << H). In configuration (**a**), the partition is open at the bottom and is partially filled with the same liquid to a level h such that the air pressure above the liquid is p₀. In configuration (**b**), the partition is sealed by a light (massless) panel such that it contains air at the same pressure p₀ as in configuration (**a**), and the gap under the partition is full of trapped air. Determine which configuration requires a partition with a *lower* minimum weight per unit width (*i.e.*, W_a or W_b) to prevent overturning.

Please briefly explain your analysis.



- 2) A liquid of constant density ρ falls uniformly with a velocity V into a section of a short horizontal rectangular open channel of width b as shown below. Neglect viscous effects and assume that the horizontal flow is uniform at each x-location.
 - a) Find an expression for the flow rate Q out of the channel.
 - **b**) Find an expression for the liquid height h_1 in terms of the liquid height h_2 , the given velocity *V*, and the length *L*.



- 3) A floating buoy of mass *m* and cross-sectional area at the free surface *A* bobs up and down with a natural frequency *f*.
 - a) Using dimensional analysis, determine how the frequency f depends on the buoy parameters, the density of the water ρ and gravitational acceleration g.
 - **b**) Then use the observation that the relevant fluid property for this problem is not the density, but rather the specific weight of the water γ , to simplify your equation for the frequency. <u>Briefly</u> explain why the specific weight is the relevant fluid property.
 - **c)** Based on your dimensional analysis results, how would you modify the design of an instrument buoy to maximize its oscillation period (to, for example, minimize disturbances to the sensitive instruments)?

4) Consider a capillary tube placed inside a pool of liquid, as shown below. Find an approximate relation for the rate at which the free surface rises, or *dh/dt*, assuming the liquid is wetting the surface of the tube (so θ ≈ 0) and the normal stress at the free surface inside the tube is 2σ/*R* where σ is the surface tension of the liquid-air interface and *R* is the radius of curvature of the liquid surface inside the capillary. The ambient pressure is atmospheric, or p_{atm}, and gravity acts along the vertical. Assume that the flow inside the capillary is governed by the simplified Navier-Stokes equation for steady and fully-developed flow inside a round pipe.

Please show all intermediate steps and state all assumptions in your derivation.

