1) A new airfoil shape is to be tested in a wind tunnel. This prototype airfoil will be used on an airplane designed to fly at a speed of $900 \mathrm{~km} / \mathrm{hr}$ at atmospheric pressure $p_{\text {atm. }}$. The designers wish to measure the drag on a $1 / 15$ scale model to determine the drag on the prototype airfoil. For this test, a wind tunnel is available which is operated at a pressure of $20 p_{\text {atm }}$ at ambient temperature.
a) What is the required velocity of the wind tunnel for the test?
b) If the drag force measured on the model is 350 N , what will be the drag on the prototype airfoil?
c) If the model airfoil creates a resonant eddy in the wind tunnel with a frequency of 60 Hz , what would the equivalent frequency be from the prototype?

Remember to state and justify all your assumptions and to define your variables.
2) A parabolic gate of profile $z=x^{2}$ (the origin of the $z$-coordinate is taken to be at the bottom) is bounded on one side by a stratified lake with a density profile $\rho(z)=\rho_{0}(1-\alpha z)$ and on the other by air at atmospheric pressure. The gate is hinged at the top and kept in a closed position by a small "stop" placed at the bottom and shown in the figure below. The free surface of the lake is also at atmospheric pressure.

a) What is the gage-pressure distribution in the stratified lake?
b) What is the force exerted on the "stop"?
3) Consider the two-dimensional incompressible flow of a Newtonian fluid of known viscosity $\mu$ and density $\rho$ through a planar contraction. The contraction has an $x$ dimension of $L$ and a half-width ( $y$-dimension) that varies linearly with $x$ :

$$
h(x)=A x+B,
$$

where $A$ and $B$ are known. The flow is driven through the contraction by an axial pressure gradient $d p / d x$ at a known volume flow rate per unit dimension normal to the page $Q$. At high Reynolds numbers, the boundary layer near the wall can be neglected, and it can be assumed that the $x$-component of velocity $u$ is only a function of $x$, or $u=u(x)$. Under this assumption, find:
a) The $x$ - and $y$-components of velocity, $u$ and $v$, respectively, in terms of the known quantities;
b) The fluid particle acceleration through the contraction; and
c) The axial pressure gradient $d p / d x$.

Please state all additional assumptions.

4) In the flow system shown in sketch a below, a piston moves to the right with constant velocity $U_{\mathrm{p}}$ within a long tube of internal diameter $D$ and length $L(L \gg D)$. The piston of length $L_{\mathrm{p}}$ is used to draw air into the tube from the left and deliver air to the right. The ambient conditions at the tube inlet and outlet are atmospheric. The piston motion within the tube is frictionless and the pressures just upstream and downstream of the piston are $p_{1}$ and $p_{2}$, respectively. It is proposed to control the motion of the piston by opening a cylindrical conduit along the piston's centerline having a small, variable diameter $d\left(d \ll D\right.$ and $\left.d \ll L_{\mathrm{p}}\right)$, as shown in sketch $\mathbf{b}$.

Using control volume analysis, determine the change in the power that is necessary to move the piston at $U_{\mathrm{p}}$ when air is flowing through the cylindrical conduit.

You may assume that:
i) the pressures at inlet and outlet of the conduit remain unchanged;
ii) the velocity distributions at the inlet and outlet sections are fully developed;
iii) the (air) friction factor within the conduit is $f$;
iv) the kinetic energy correction factor is $\alpha$; and
v) the inlet and outlet losses of the conduit are negligible.

Based on your answer, briefly explain what happens if $d$ becomes larger (but is still smaller than $D$ ).
a

b


