GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Semester 2003

Fluid Mechanics
EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME) --

- **Please sign your name on the back of this page—**

1) In the study of pulsatile, or periodic, blood flow through arteries, it is useful to scale up the experimental models to improve the spatial and temporal resolution of the measurements. In a medium-sized artery, the diameter D = 6 mm, the absolute viscosity and density of blood at an absolute temperature T = 310 K are μ = 3.5 cP (centipoise) and ρ = 1 g/cm^3, respectively, and the average velocity V = 10 cm^3/s. The pulsatile flow frequency f = 1 Hz.

a) Using dimensional analysis, determine the important independent nondimensional groups for this flow.

b) For a large-scale experimental test model of this artery with a diameter of 30 mm, what would be the resultant average velocity in the model V_m assuming the test fluid is water at T_m = 298 K with a viscosity of 1 cP?

c) Determine the periodic frequency in this model f_m.

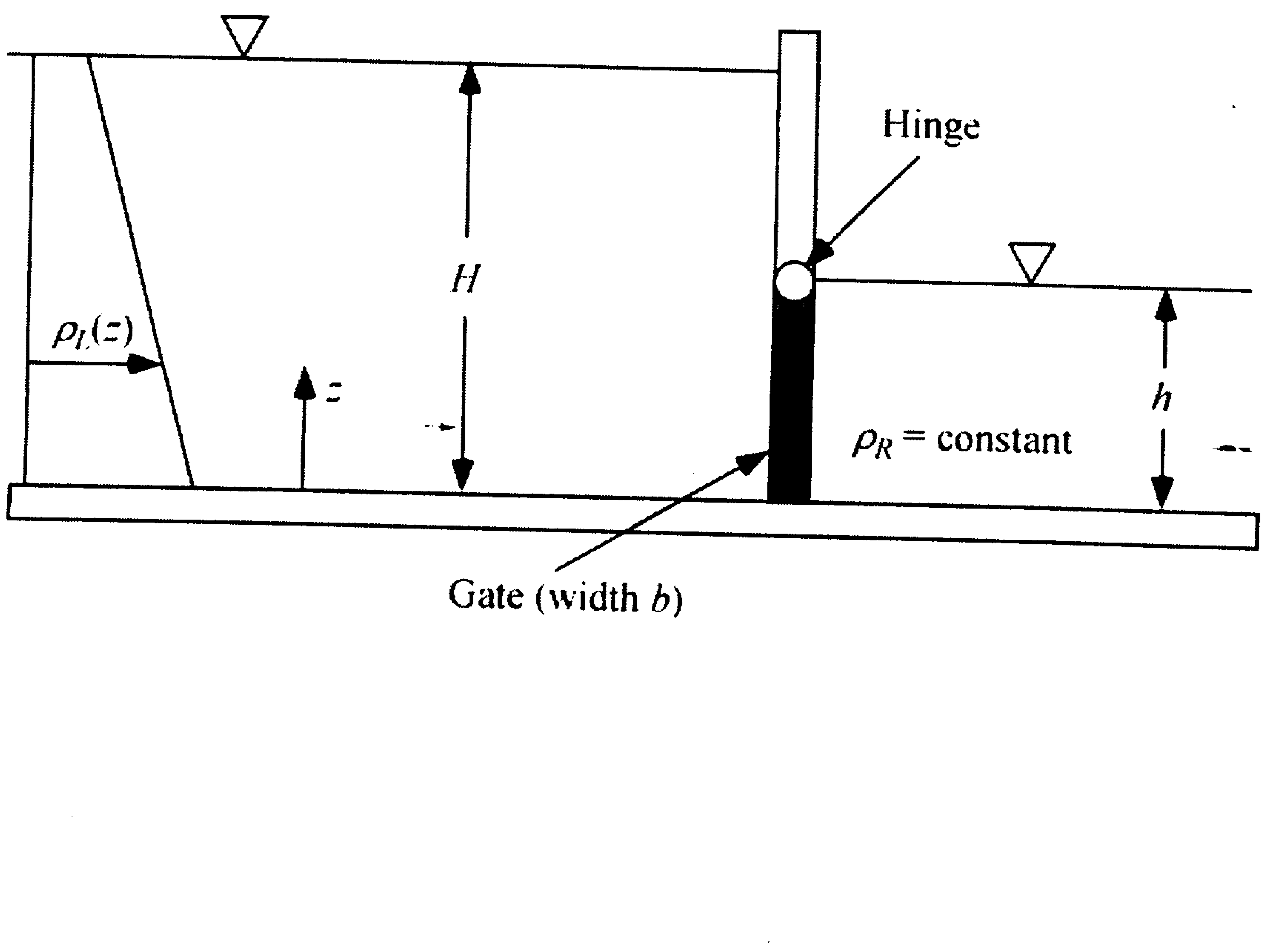
d) Flow diagnostic instruments such as laser-Doppler velocimeters (LDVs) often have a preferred resolution range. How could you improve the experiments to increase the average velocity and periodic frequency for this large-scale model? Given that LDVs are optical diagnostic instruments and therefore require optical access to the test model, what is an additional consideration in setting up this experiment?

2) As shown below, a vertical wall separates two bodies of liquid. The liquid on the left side of the wall is *linearly stratified*, with a density that varies as $\rho_L(z) = \rho^*(1-\alpha z)$, where ρ^* is the density at the bottom wall, $\alpha > 0$ is a constant and z is oriented vertically upward with origin at the bottom. The liquid on the right-hand side of the wall is of constant density ρ_R. In the lower portion of the vertical wall is a gate of height h and width b (normal to the figure), hinged at the top, permitting it to rotate either to the left or right as viewed in the figure.

a) What is the *gage*-pressure distribution (i.e., as a function of z) in the liquid on the left-hand side of the wall?

b) What is the *force* that the liquid on the *left-hand side* of the wall exerts upon the *gate*?

c) If the liquid on the right-hand side of the wall is only as deep as the gate itself (as shown in the figure), what density ρ_R would be necessary to keep the gate from opening in either direction?



3) Consider two-dimensional "squeeze film flow", where a plate with an x-extent of $2L$ starts to move downwards at time $t = 0$ at a constant speed V towards a stationary flat wall. The gap between the plate and the wall is completely filled with a liquid of constant density and viscosity ρ and μ, respectively, and the gap height at time t is denoted by $h(t)$. As the plate moves downwards, it squeezes out liquid around its edge. The velocity at the edge of the plate has only a component along x and is parabolic in z:

$$u(z) = U\left[\frac{z}{h(t)} - \left(\frac{z}{h(t)}\right)^2\right].$$

You are given L, V, ρ, μ and the initial gap thickness h_o.

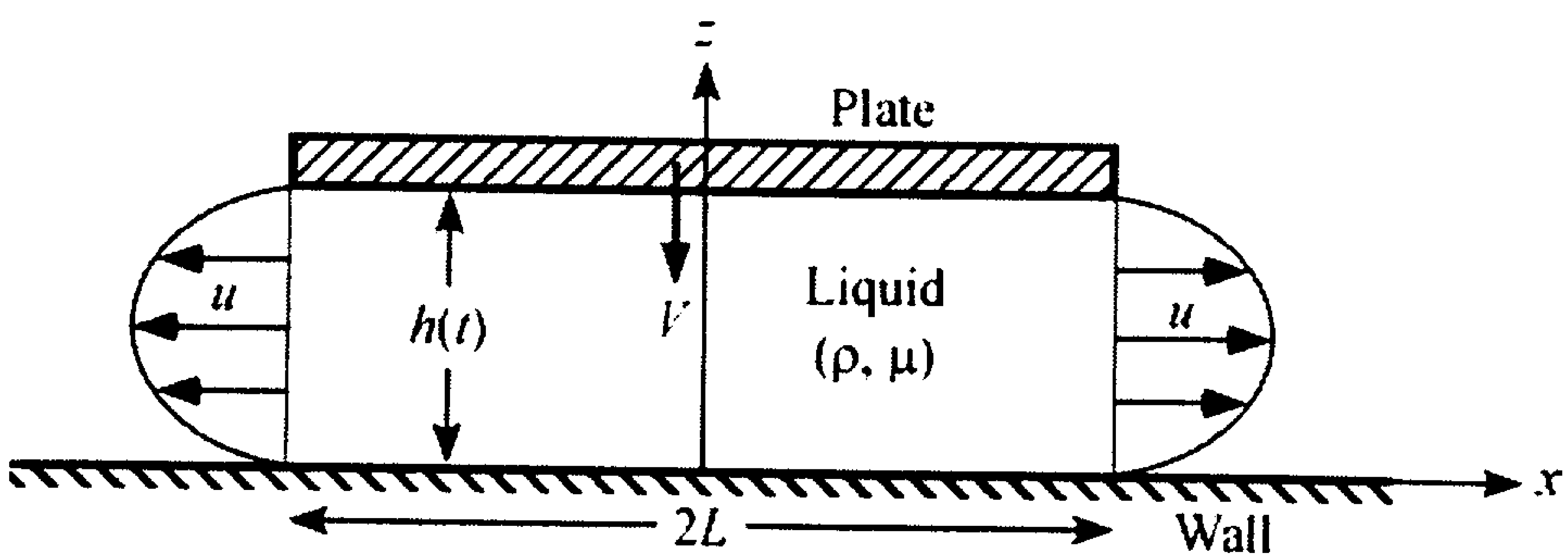
a) Find $h(t)$.

b) Determine the maximum velocity at the edges, or U_{max}. What happens to U_{max} as the film becomes thin (*i.e.*, $h \to 0$)? Does this physically make sense? Explain (please be brief)!

c) In the limit of creeping or Stokes flow (*i.e.*, zero Reynolds number), the pressure field in this flow is given by a simplified form of Reynolds' lubrication equation:

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) = 12\mu V$$

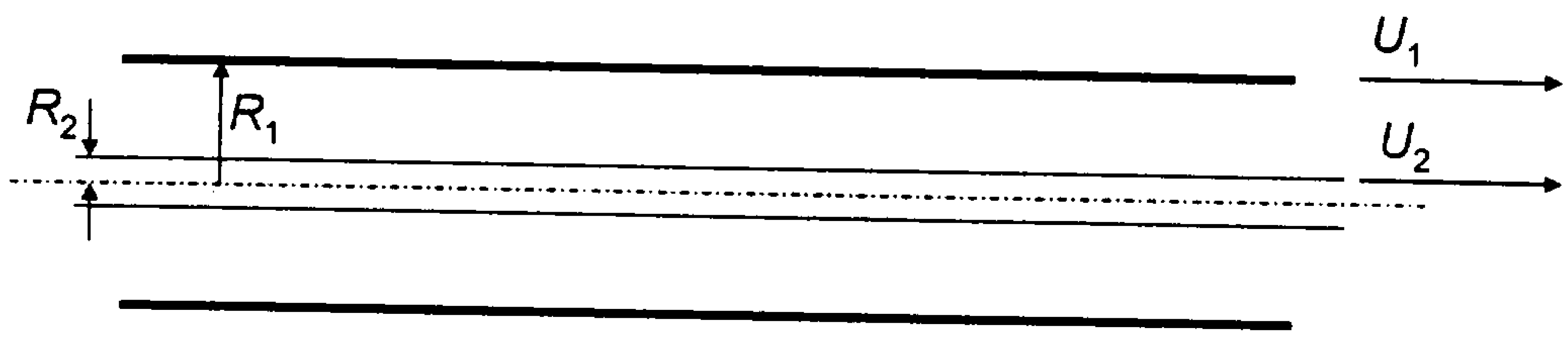
Nondimensionalize this equation to determine the characteristic scale for the pressure (you are NOT expected to solve the equation). Based upon this scale, what happens to the pressure in the squeeze film as the film becomes thin?



4) The annular gap between two long, concentric horizontal tubes is filled with fluid having constant viscosity μ and constant density ρ. Each of the tubes has a circular cross section and, as shown in the sketch below, the internal radius of the outer tube is R_1 while the outer radius of the center tube is R_2 ($R_1 - R_2$ is small compared to the length of the tubes). In the absence of an axial pressure gradient, it is desired to set the fluid within the gap into motion by moving either the outer or inner tube at constant velocity U to the right as shown in the sketch while keeping the other tube stationary (i.e., $U_1 = U$ and $U_2 = 0$, or $U_1 = 0$ and $U_2 = U$).

a) Determine the velocity distributions that are induced within the annular gap (far from its downstream or upstream ends) a long time after the beginning of the motion of the outer and inner tubes (while in each case the other tube is at rest).

b) Neglecting end effects, determine the force that is exerted in each case on the (inner or outer) stationary tube when the other tube is moving (with constant velocity U). Are these forces the same? *Explain briefly.*



Continuity

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r \upsilon_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho \upsilon_\theta) + \frac{\partial}{\partial z}(\rho \upsilon_z) = 0$$

Navier Stokes Equations

$$\rho\left(\frac{\partial \upsilon_r}{\partial t} + \upsilon_r \frac{\partial \upsilon_r}{\partial r} + \frac{\upsilon_\theta}{r}\frac{\partial \upsilon_r}{\partial \theta} - \frac{\upsilon_\theta^2}{r} + \upsilon_z \frac{\partial \upsilon_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\upsilon_r)\right) + \frac{1}{r^2}\frac{\partial^2 \upsilon_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial \upsilon_\theta}{\partial \theta} + \frac{\partial^2 \upsilon_r}{\partial z^2}\right] + \rho g_r$$

$$\rho\left(\frac{\partial \upsilon_\theta}{\partial t} + \upsilon_r \frac{\partial \upsilon_\theta}{\partial r} + \frac{\upsilon_\theta}{r}\frac{\partial \upsilon_\theta}{\partial \theta} + \frac{\upsilon_r \upsilon_\theta}{r} + \upsilon_z \frac{\partial \upsilon_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\upsilon_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 \upsilon_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial \upsilon_r}{\partial \theta} + \frac{\partial^2 \upsilon_\theta}{\partial z^2}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial \upsilon_z}{\partial t} + \upsilon_r \frac{\partial \upsilon_z}{\partial r} + \frac{\upsilon_\theta}{r}\frac{\partial \upsilon_z}{\partial \theta} + \upsilon_z \frac{\partial \upsilon_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \upsilon_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \upsilon_z}{\partial \theta^2} + \frac{\partial^2 \upsilon_z}{\partial z^2}\right] + \rho g_z$$