GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam –Fall Semester 2010

FLUID MECHANICS

EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

✱ Please sign your **name** on the back of this page —

Problem 1

Consider gravity-driven flow down an incline of angle θ with respect to horizontal. A bi-layer of two fluids, each of distinct viscosity (μ_1, μ_2) and density (ρ_1, ρ_2) flows down the incline at steady-state. Each layer has a height of *H*.

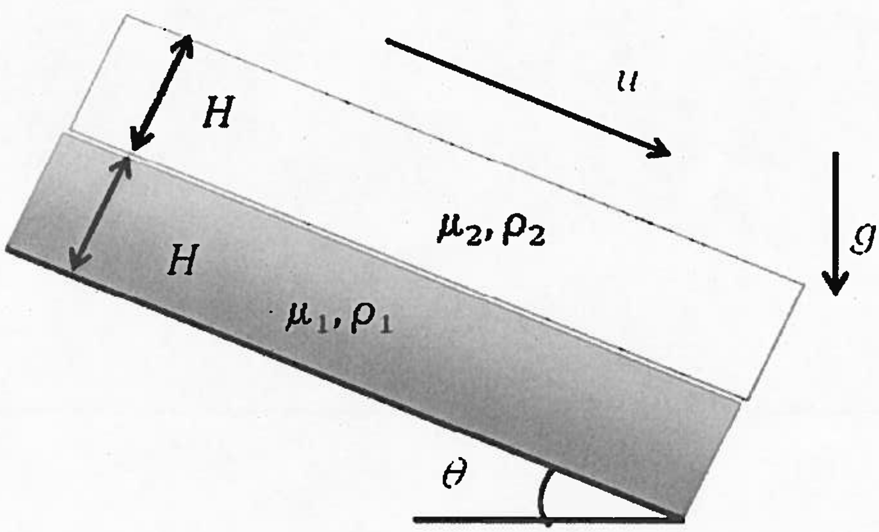
a. Select appropriate coordinate axes.
b. Use the Navier-Stokes equations to determine the governing equations in each fluid layer. State what approximations are used (e.g, steady-state, 2-D flow)
c. Provide sufficient boundary conditions and solve for the velocity flow profile in each layer.
d. What is the total volumetric flow rate Q down the incline?

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+g_x$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+g_y$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+g_z$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$



Problem 2

An oil spill accident takes place in an ocean A at the depth of 1000 m. Oil leaks from a well with a diameter $D = 10\ cm$ with a constant mass flow rate $\dot{m}$. Engineers from an oil company B analyzed underwater photographical pictures of the oil jet just above the well riser and found that the jet disperses and oil forms stable droplets with a diameter $d \approx 5\ mm$. The oil and water densities are $\rho_o = 850\,kg/m^3$ and $\rho_w = 1000\,kg/m^3$, respectively, and interfacial tension between water and oil is $\sigma = 0.03\,N/m$. The fluid properties can be assumed to be independent of depth.

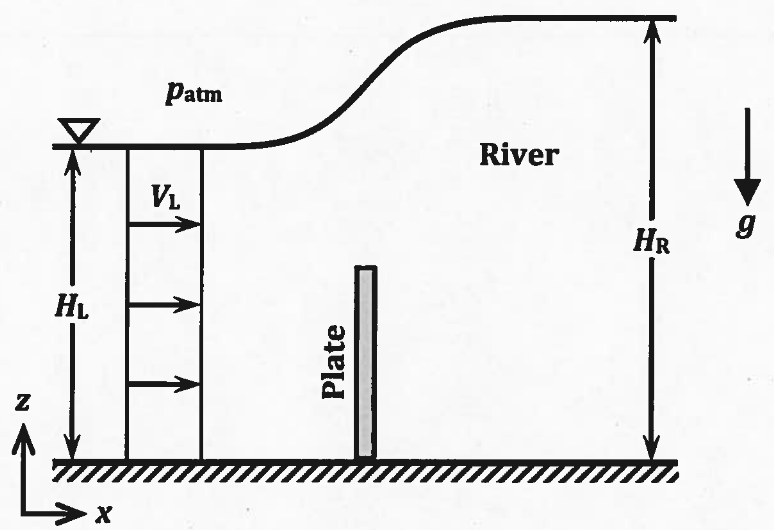
- Define the nondimensional groups relevant for this problem and explain their physical meaning.
- Using dimensional analysis estimate the velocity of droplets V, in m/s, and the mass flow rate $\dot{m}$ in the well, in kg/s. (Hint: droplet breaks up if the force due to fluid flow exceeds the force due to surface tension.)
- Given that the gravitational acceleration $g = 10\,m/s$, estimate the rising velocity V_r of droplets when they are far from the ground. Assume that the drag coefficient $C_D \sim 1$. How does this velocity vary with depth? Explain.
- Will the diameter of droplets change when they travel with velocity V_r? Explain.

Problem 3

A vertical plate is completely submerged in a river, and attached to the horizontal bottom of the river. The river has a depth H_L and a speed V_L to the left of the plate, and a depth H_R to the right of the plate where $H_R > H_L$, as shown below. The free surface of the river is at atmospheric pressure, p_{atm}. Assume that:

- the flow is steady
- the velocities are uniform to the left and the right of the plate
- friction between the river and its bottom is negligible
- the density of the water in the river, ρ, is constant
- the dimension of the river normal to the page is unity.

Define your control volume.



a) What is the speed of the river to the right of the plate, V_R?

b) What is the horizontal (x) component of the force exerted by the flowing river on the plate, $\vec{F}_x$?

c) For the river to flow to the right as shown, how must V_L depend upon the other known parameters? Please justify your answer.

Problem 4

It is proposed to construct a research buoy equipped with various environmental sensors that will be released from a small submarine at a depth H below the surface of a deep, fresh water lake as shown in the sketch below (the water density, viscosity and temperature, ρ, μ, and T, respectively, are invariant with depth). The spherical balloon (diameter $d << H$) is fabricated using thin elastic polymeric material having negligible weight, and is typically filled with air (at pressure p_o, and the water temperature T) to provide buoyancy force (the air may be thought of as ideal gas). The weight of the sensors package (including the connecting cable) is W, and it may be assumed that its buoyancy is negligible compared to the buoyancy of the balloon. For the purpose of the measurements, it is required that the buoy will ascend or descend at a constant, low speed V at which the drag force on the buoy D is assumed to be negligible. To this end, the buoy's release mechanism is designed to provide an initial upward or downward speed V.

a) Determine the diameter of the balloon d during ascent after the buoy is released with upward speed V. Discuss briefly the principles and limitations (if any) of the proposed design.

b) Explain briefly whether there are any fundamental differences between the buoy's ascent and descent (assuming that it is released with speed V in either direction)?

c) Would you suggest any changes in the proposed design? Explain briefly.