1) Consider an ideal gas, whose equation of state is given by $p = \rho R T$, where p is the absolute pressure, ρ is the density, R is the gas constant and T is the absolute temperature. Assume that the temperature distribution in the ideal gas decreases linearly with elevation (z is positive upwards) as $T(z) = T_a - \beta z$, where $\beta > 0$ is a constant.

   a) Derive an expression for the ***pressure distribution*** $p(z)$ for $z \geq 0$ if $p(0) = p_a$ at the surface ($z = 0$).

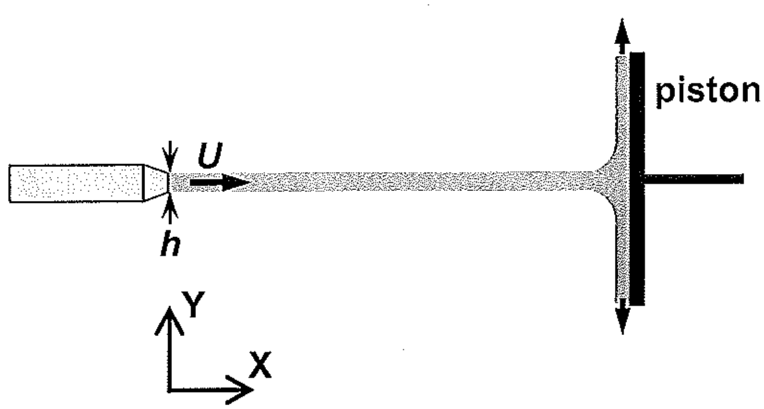
   b) At what altitude z^* would the pressure be equal to half its value at the surface?

2) A rectangular jet of height h and width w of a liquid of constant density ρ is moving with a constant velocity U (relative to the stationary frame of reference X-Y) and impinges on a stationary piston as shown schematically in the figure below. The jet fluid spreads symmetrically along the surface of the plate in the Y-Z plane and forms two secondary jets. Assume that:

- Viscous effects along the surface are negligible so that the velocity of each liquid jets is uniform in the X-Y plane
- Gravitational effects (along the Z direction normal to the page) are negligible so that each jet is effectively two-dimensional
- Surface tension effects are negligible
- The pressure near the nozzle and the piston surface is atmospheric.

a) Using a ***stationary control volume***, determine the magnitude and direction of the force $\bar{F}_p$ that should be applied to piston so that it moves to the ***left*** with constant velocity U_p.

b) Based on your analysis, ***briefly*** discuss how $\bar{F}_p$ affects the momentum flux of each of the secondary jets compared to that for the case of a stationary piston.



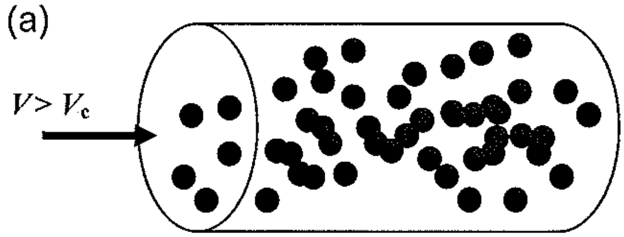
3) You are asked to develop a small model experiment to find the critical fluid velocity, V_c, required to keep spherical particles suspended in an incompressible fluid (*i.e.*, fluidized) inside a horizontal circular tube, as shown in Figure (a). When the fluid velocity V is less than V_c, the particles accumulate at the bottom of the tube, as shown in Figure (b). Assume that the particles are spherical solid particles of diameter d and density ρ_s. The diameter of the pipe is D.

a) Using dimensional analysis, what are the variables that determine the critical velocity V_c?

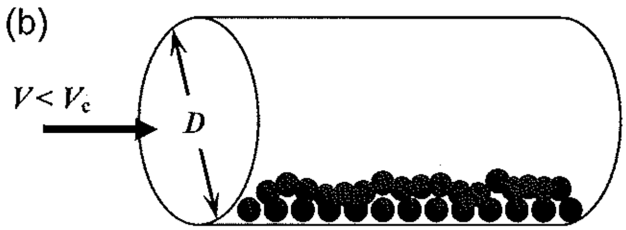
b) Determine what is required for a model experiment to be similar to a full-scale prototype, and the relationship between the critical velocity for the model V_c^m and that for the prototype.

c) If the model is 1/5th the scale of the prototype and the model and prototype use fluids of the same density, what is the V_c^m required for the model and the prototype to be similar?

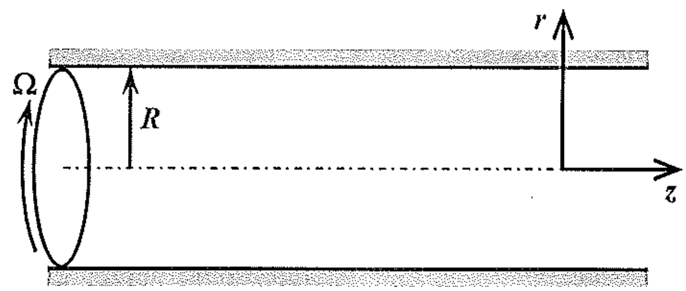
(a)



(b)



4) Consider the steady, fully developed and incompressible flow of a Newtonian fluid (density ρ, viscosity μ) driven by a constant axial pressure gradient $\Delta p / L$ (note that $\Delta p > 0$) through a horizontal round pipe of inner radius R. The pipe is rotating about its axis at a constant angular speed Ω. Assume that body forces are negligible, and that the pressure at the origin is p_o.



a) What are the boundary conditions for this flow on the velocity field $\vec{\mathbf{V}} = V_r\hat{\mathbf{r}} + V_\theta\hat{\boldsymbol{\theta}} + V_z\hat{\mathbf{z}}$?

b) Using the incompressible Continuity and Navier-Stokes equations in cylindrical polar coordinates shown below, determine the velocity field $\vec{\mathbf{V}}$ and the pressure field p for this flow. Please clearly state all ***additional*** assumptions.

<u>Incompressible Continuity:</u>

$$\frac{1}{r}\frac{\partial}{\partial r}(rV_r) + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z} = 0$$

<u>Navier-Stokes:</u>

$$\rho\left(\frac{\partial V_r}{\partial t} + V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} - \frac{V_\theta^2}{r} + V_z\frac{\partial V_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \rho f_r + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rV_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2} + \frac{\partial^2 V_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta}\right] \quad (r)$$

$$\rho\left(\frac{\partial V_\theta}{\partial t} + V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{V_r V_\theta}{r} + V_z\frac{\partial V_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \rho f_\theta + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rV_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2} + \frac{\partial^2 V_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial V_r}{\partial \theta}\right] \quad (\theta)$$

$$\rho\left(\frac{\partial V_z}{\partial t} + V_r\frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta} + V_z\frac{\partial V_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho f_z + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2} + \frac{\partial^2 V_z}{\partial z^2}\right] \quad (z)$$