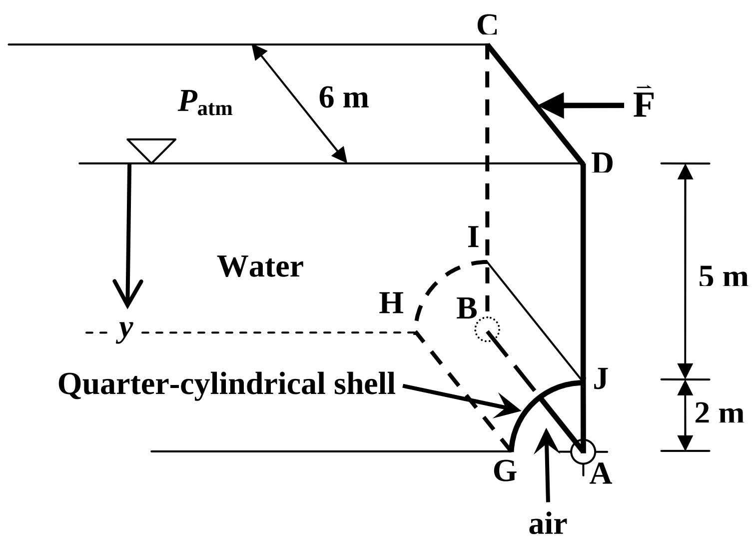
1) Consider the rectangular gate ABCD shown below. The gate is connected to a hinge at AB and attached to the quarter-(circular)cylindrical shell IJGH of the same material at IJ. The gate ABCD and quarter-cylinder IJGH are free to rotate about the hinge AB. Note that there is an air-filled gap between the gate and the shell. All dimensions are given in the diagram.

The liquid in the tank is stratified such that the specific weight is given by $\gamma = \gamma_0(1+0.03y)$, where γ_0, the specific weight of water at the top of the tank ($y = 0$) is 9.8 kN/m^3. The weight and thickness of the gate and the quarter-cylinder can be neglected. *Clearly state any additional assumptions you make to solve this problem.*

a) Determine the horizontal component of the force on the quarter-cylindrical shell IJGH.

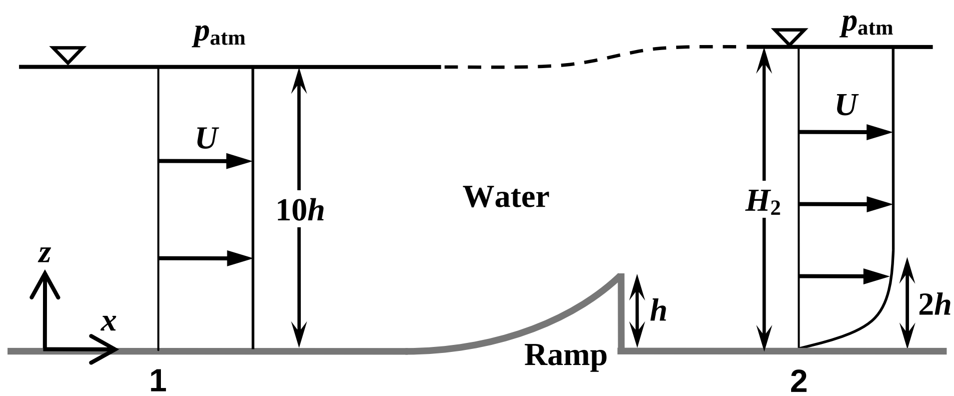
b) Calculate the force $\vec{\mathbf{F}}$ at the top of the gate required to keep the gate just closed.



2) Water (of constant density ρ) flows steadily in a river whose free surface is at atmospheric pressure p_{atm}. A "ramp" of height h on the bottom of the river disturbs the flow. Upstream of the ramp at location **1**, the velocity profile is uniform with a speed U over a depth $H_1 = 10h$. Downstream of the ramp at location **2**, the velocity profile is:

$$\frac{u_2(z)}{U} = \begin{cases} 1 & \text{for } \delta \le z \le H_2 \\ \dfrac{3}{2}\dfrac{y}{\delta} - \dfrac{1}{2}\left(\dfrac{y}{\delta}\right)^3 & \text{for } 0 \le z \le \delta \end{cases}$$

where $\delta = 2h$ and H_2 is the depth of the river at location **2**. What is H_2? What is $\vec{\mathbf{F}}_x$, the horizontal (x) force exerted by the river on the ramp (per unit dimension normal to the page)? Neglect any viscous forces on the river bottom, and assume that the flow in the river is two-dimensional.

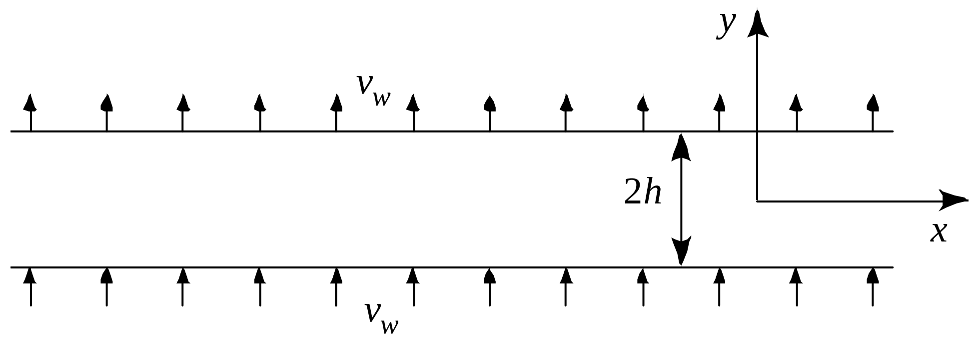


3) A nuclear explosion results in an instantaneous release of energy *E* within a small spatial domain and produces a spherical shock wave of radius *R*, where the pressure within the shock wave is much greater that the initial (undisturbed) air pressure.

a) Using dimensional analysis, determine how the shock wave radius *R* depends on the time *t*, the energy release *E*, and on the density ρ_o of the undisturbed air. Identify unknown constants in your expression and explain how they could be determined.

b) Would the radius of the shock wave grow faster with time if the explosion is set off at an altitude of 30,000 ft compared to an explosion at ground level? ***Explain briefly.***

4) A pressure-driven flow of an incompressible fluid exists between two horizontal porous plates spaced a distance $2h$ apart as shown below. Fluid is also injected from the lower plate into the gap between the plates at a speed v_w and sucked out of the gap into the top plate at the same speed. The flow is steady and fully developed, with negligible body forces.



a) Use the Navier-Stokes equations for two-dimensional flow shown below to find ordinary differential equations (ODEs) for the horizontal and vertical velocity profiles in the gap. Include appropriate boundary conditions.

$$\rho\{u_t + uu_x + vu_y\} = -p_x + \mu\{u_{xx} + u_{yy}\}$$
$$\rho\{v_t + uv_x + vv_y\} = -p_y + \mu\{v_{xx} + v_{yy}\}$$
$$u_x + v_y = 0$$

b) Scale the ODE for the horizontal velocity using the velocity scale $U_s = -\dfrac{p_x h^2}{2\mu}$, where p_x is the constant pressure gradient driving the flow. Your final equation should have one parameter given by the Reynolds number $Re = \dfrac{\rho v_w h}{\mu}$.

c) Solve your equation for the horizontal velocity.

d) Sketch the horizontal velocity profile for small and large values of Re.