1) Flooding in New Orleans can be modeled using dynamic similarity, even with distorted scales. Similarity can be used to create distorted models that exhibit fluid behavior with different time scales than the actual situation. Consider a model of the city of New Orleans built to a horizontal scale ratio of 1:5000 (model:prototype) and a vertical scale of 1:100.

You are given the following parameters; assume the working fluid is also water for the model.

<u>New Orleans</u>:

Average velocity $U = 2$ cm/s
Horizontal length scale $L = 50$ m
Vertical length scale $H = 1$ m
Gravitational constant $g = 9.8$ m/s^2
Dynamic viscosity of water $\mu = 1$ cSt
Time scale $T = 1$ day

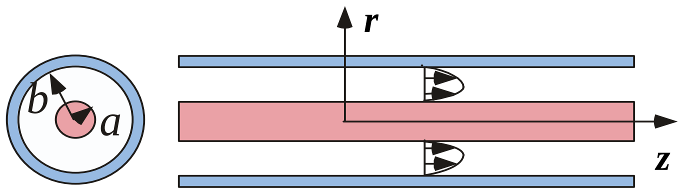
<u>Model</u>:

Horizontal length scale $L_M = 1$ cm
Vertical length scale $H_M = 1$ cm

a) Determine the relevant non-dimensional groups, or Π terms. You will find that the relevant model scales may vary among different Π terms because of the geometric distortion. Justify your selection of scales based upon their relevance to the important effects (*e.g.*, Which length scale is the relevant one for hydrostatic pressure? Which length scale is relevant for viscous forces?)

b) Based upon your answer to part (a), determine the time scale for the model that corresponds to one day in New Orleans.

2) A Newtonian fluid of constant density ρ and viscosity μ is driven through the annular gap between two concentric cylinders of radii a and b, as shown below, by a constant axial pressure gradient $dp/dz = -K$, where $K > 0$.



a) Simplify the equations given below for the case of steady (S), fully developed (F), non-swirling (NS), rotationally symmetric (R) flow in the absence of body forces (B) and without an azimuthal (θ) pressure gradient. Apply the assumptions in the order listed, and use the letter(s) indicated above to denote the assumption used to eliminate each term. *Prove* that the radial component of velocity must vanish.

b) State the appropriate boundary conditions for viscous flow.

c) Determine the solution to the problem based upon the simplification in part (a) and the boundary conditions in part (b).

The Navier-Stokes and Continuity equations, in cylindrical coordinates, are:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta{}^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] - \frac{\partial p}{\partial r} + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] - \frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_z,$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0.$$

3) A round tube of constant cross-sectional area A is partially filled with a liquid of constant density ρ and submerged in a container of the same liquid so that its bottom end is a distance a below the free surface, as shown in the sketch below. The top end of the tube is capped and the pressure of the air above the liquid surface is p_t. At time $t = 0$, air is let into the tube through a valve in the top cap, and the liquid column inside the tube begins to drain through its submerged lower end, **e**.

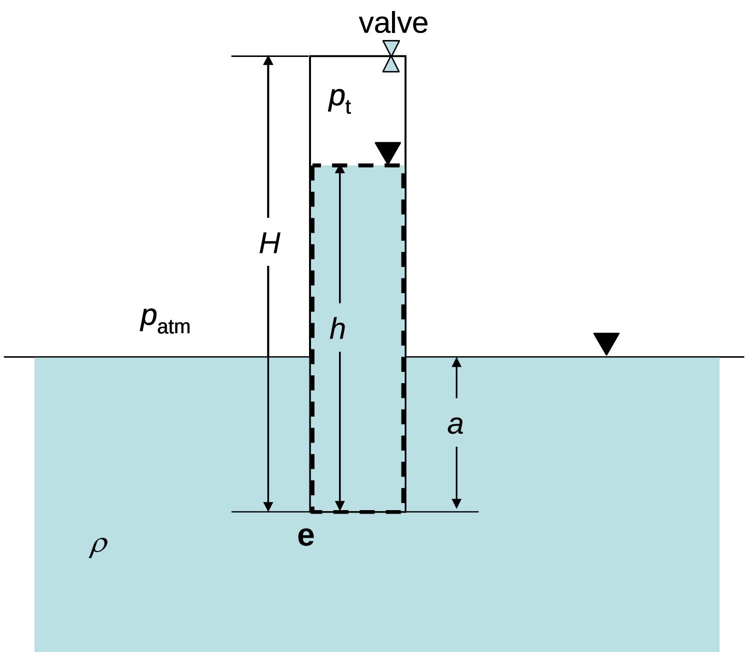
Assume that:

i. the transients due to the startup of the draining process die out very rapidly;

ii. the liquid velocity at **e,** U_e, is uniform and the pressure at **e** is the local static pressure;

iii. the airflow through the valve is regulated so that the liquid velocity at **e** remains constant over time;

iv. viscous effects along the inner surface of the tube and secondary flows inside the tube are negligible.



Using the ***fixed*** control volume denoted by the dashed line, determine the following quantities as a function of the given parameters (p_t, p_{atm}, A, a, H, ρ, g). ***Please list all your assumptions.***

a) The height of the liquid column, h, within the tube (relative to the bottom end) as a function of time t. What is the ***final*** value of h?

b) The time rate of change of the momentum of the liquid column within the tube. Can the momentum of the liquid column change with time if the liquid velocity at **e** is constant over time?

c) The magnitude of p_t.

4) Liquid of specific gravity SG = 1.2 is stored under a hemispherical dome of radius R, as shown in the figure below. This dome has a small vent hole at the top, and is attached to the bottom plate with 10 bolts. The dome is made of a material with a density twice that of water, and its wall thickness $t_s = 0.001R$. Neglect any leakage at the seam between the dome and the plate.

a) For R= 10 m, are the bolts required to keep the liquid under the dome? If so, calculate the force on each bolt and determine if the bolts are in compression or tension.

b) Answer the same questions as in part (a) is the dome containing this liquid is completely immersed in water so that the bottom plate is 11 m below the free surface of the water.

Notes:

- The volume of a sphere is $4\pi R^3/3$, and the surface area of a sphere is $4\pi R^2$
- The specific weight of water γ = 9.8 kN/m^3

Please list all your assumptions, and justify them as required.