GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Semester 2002

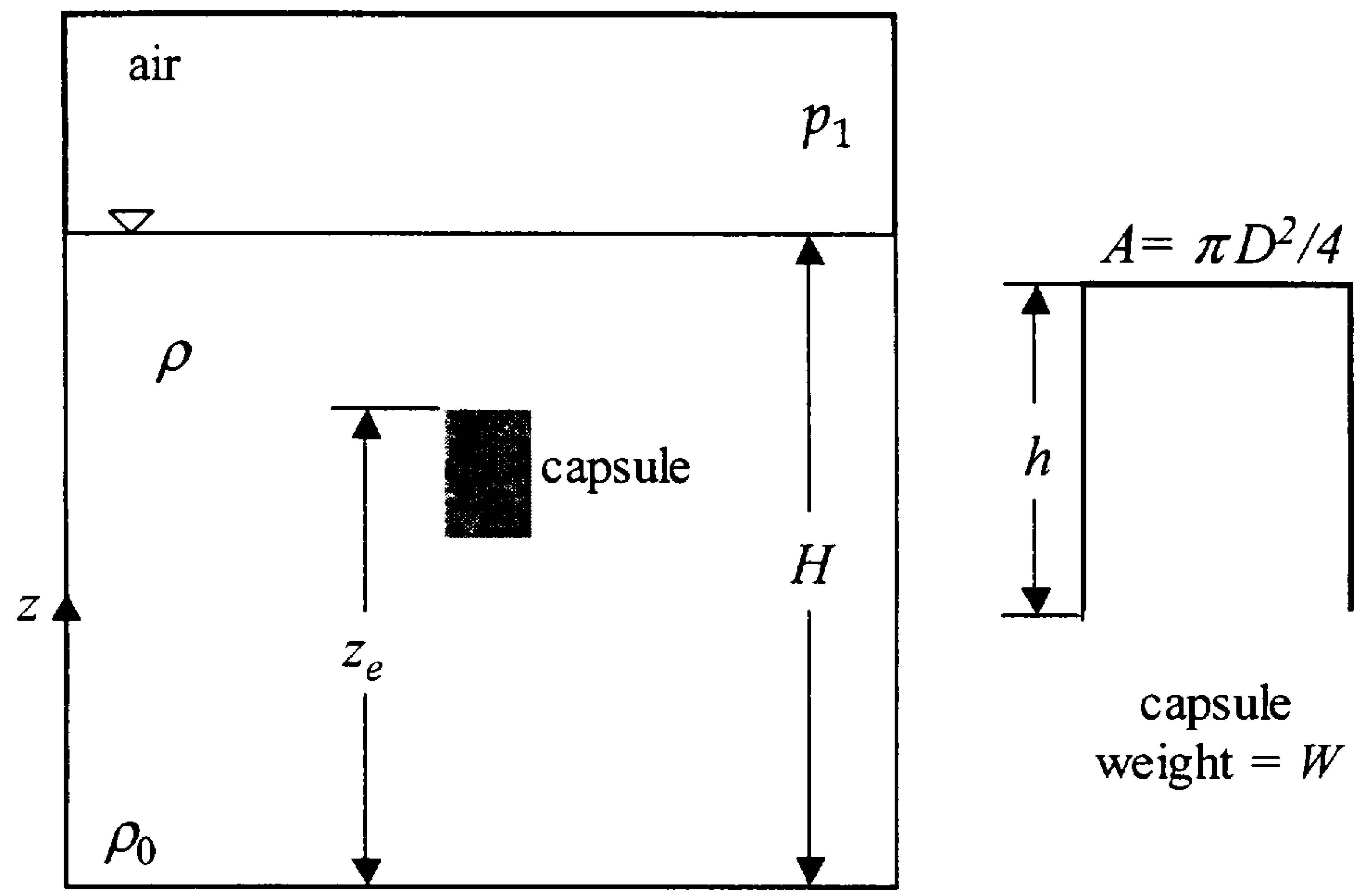
Fluid Mechanics
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

1) It is proposed to measure the pressure within a pressurized liquid tank using a suspended cylindrical capsule as shown in the sketch below. The density of the liquid $\rho(z)$ is known and increases linearly with depth ($d\rho / dz = a$, where a is a constant, and $\rho = \rho_0$ at the bottom of the tank). The airspace above the liquid in the pressurized tank is at an absolute pressure of p_1.

The capsule is a cylindrical shell (of weight W) which is open at one end. The capsule contains air at atmospheric pressure p_{atm} before it is submerged (vertically, with its open end at the bottom) in the tank. The temperatures of the air and the liquid within the tank are uniform and constant, and the capsule remains upright while it is submerged.



a) Comment on how this approach for measuring p might work, and determine the relationship between the elevation of the capsule z_e and the pressure p.

b) Will this approach work if the density *decreases* with depth? Explain briefly.

2) A raft floats on a river with an unknown constant velocity U_r. The raft is a solid flat plate of mass m, length (x-dimension) L and width (dimension normal to the page) b. Far from the raft, the river has a constant uniform velocity U_∞. Just below the raft, however, there exists a boundary layer of thickness $\delta(x)$ with a velocity profile with respect to the raft:

$$u(x,y) = (U_r - U_\infty)\left[2\frac{y}{\delta(x)} - \left(\frac{y}{\delta(x)}\right)^2\right] \qquad \text{for } 0 \le y \le \delta(x)$$

The boundary layer thickness

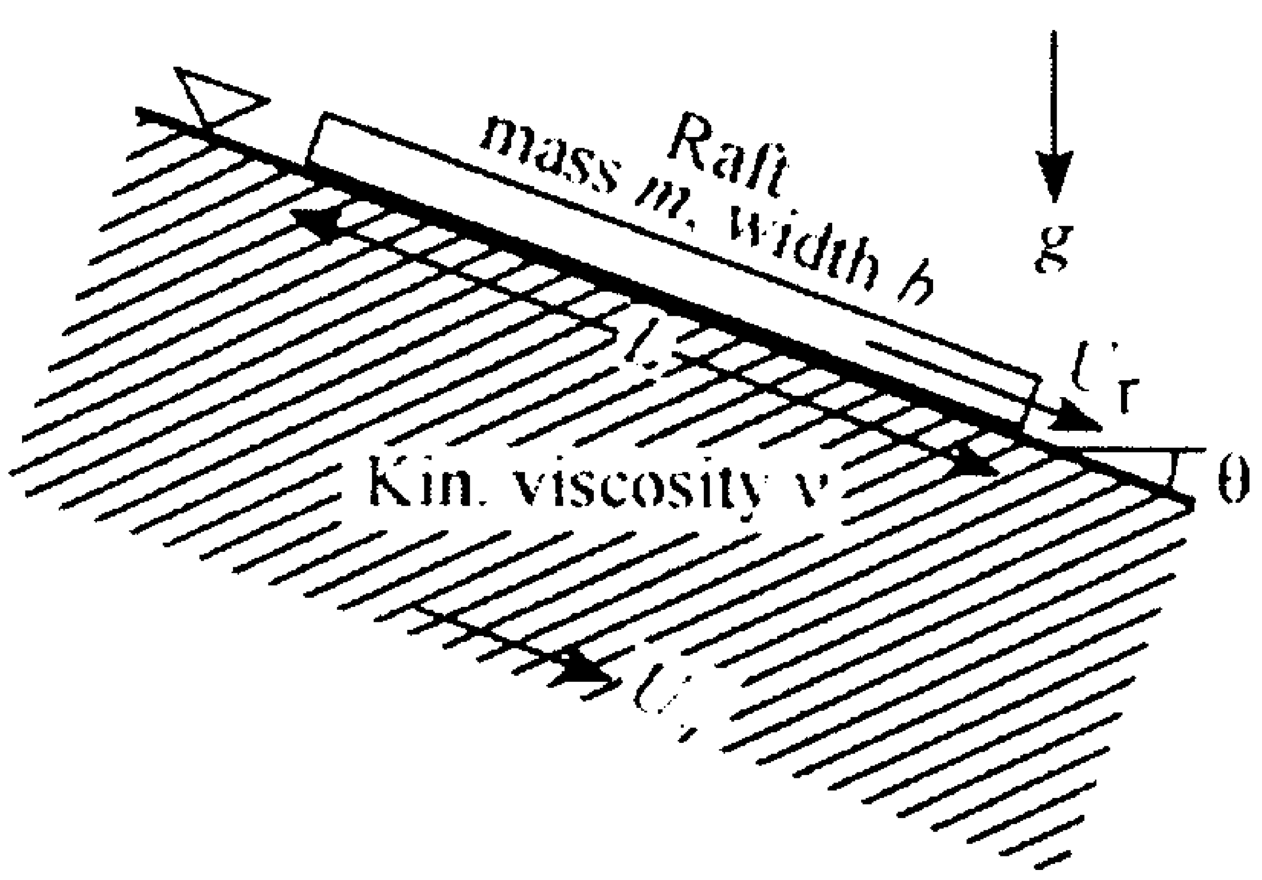
$$\delta(x) = \sqrt{\frac{30\mu}{\rho(U_r - U_\infty)}x},$$

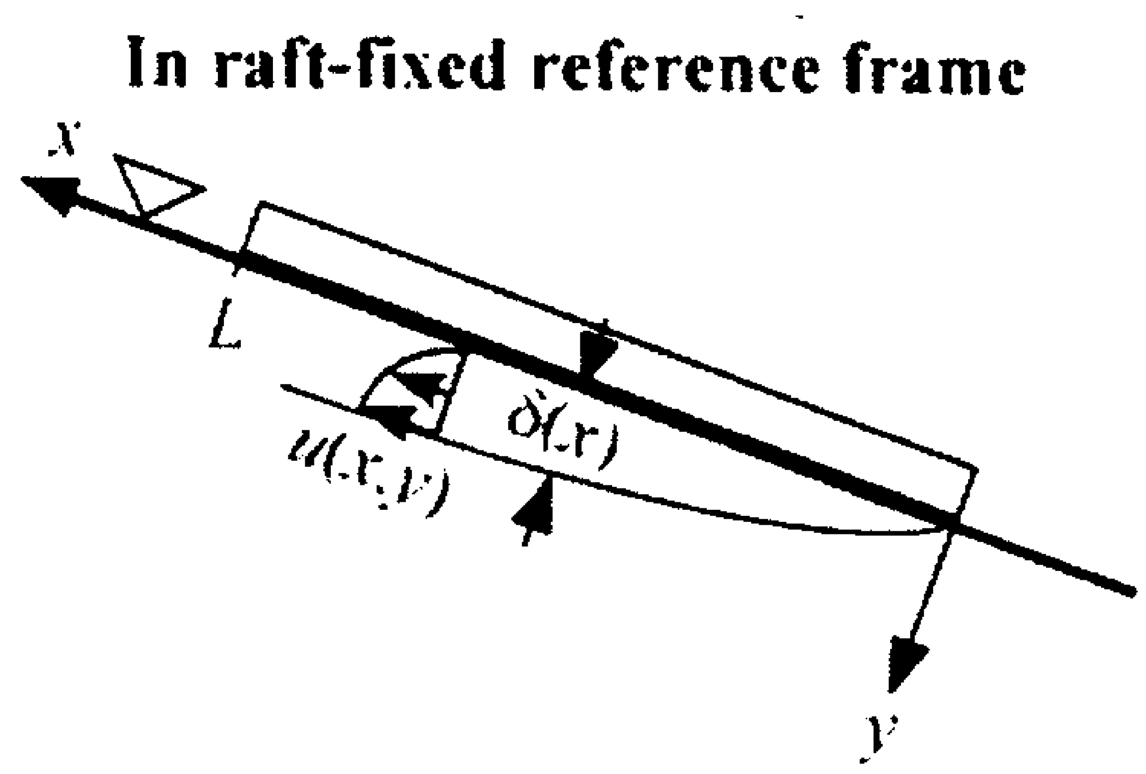
where ρ and μ are the density and absolute viscosity of the water in the river. At this location, the river free surface slopes downwards at an angle θ below the horizontal.

a) Find the friction drag on the raft in terms of U_r if only the bottom of the raft contacts the water. What is the important dimensionless group in your result?

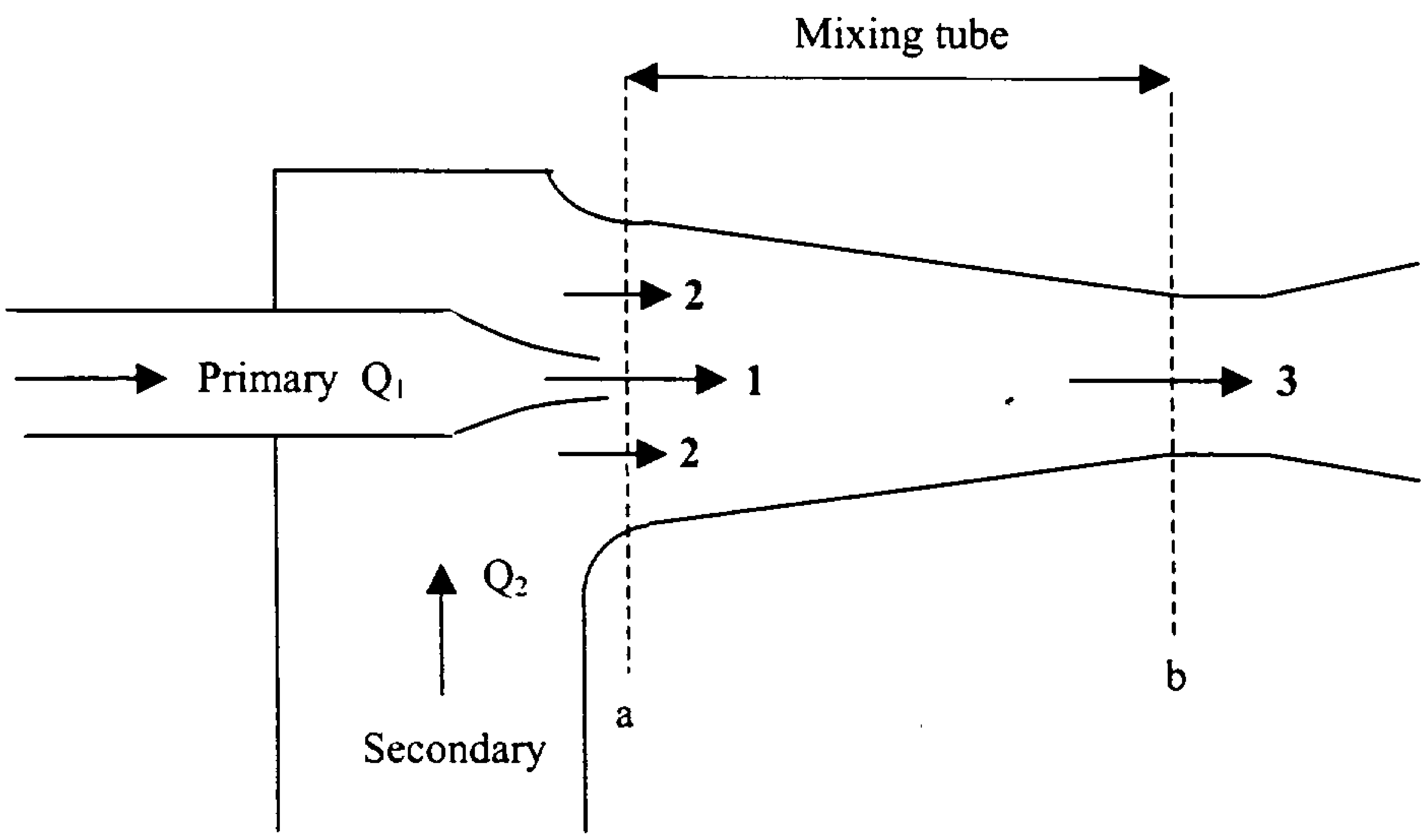
b) Determine U_r using a force balance on the raft in terms of the known parameters (m, L, b, U_∞, ν, θ, g). Under what conditions would the "slip velocity" between the raft and the river $U_r - U_\infty = 0$?

c) Physically, explain why the raft and a fluid particle in the river (far away from the raft) can have different constant speeds when both are driven by the same gravitational force per unit mass. Please be brief.





3)



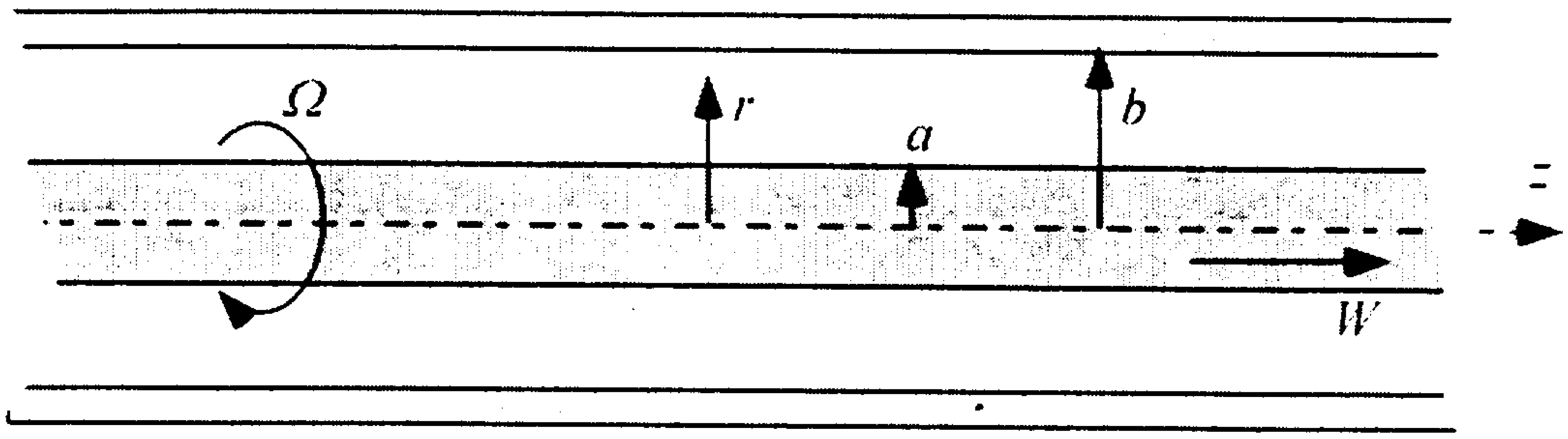
The sketch shows the horizontal mixing tube of a jet pump in which a primary high-velocity water stream entering the mixing tube at section 1 entrains a secondary low-velocity water stream entering at section 2.

As a result of turbulent mixing, the combined streams, completely mixed, leave at section 3. This particular mixing tube is designed so that there is negligible pressure drop between locations a and b, and the static pressure is constant along the walls between a and b.

a) Assuming that V_1 = 100 ft/sec; A_1 = 0.001 ft^2; the annulus area A_2 = 0.1 ft^2; Q_2 = 1 ft^3/sec and that friction on the tube walls between a and b is negligible, calculate the cross-sectional area and the velocity at the exit of the mixing tube, section 3.

b) Determine the rate of lost mechanical energy for a fluid particle flowing from the high-velocity stream at section 1 to the exit at section 3, and that for a particle flowing from the low-velocity stream at section 2 to section 3.

4)



An infinitely long annulus $a \le r \le b$ is filled with a viscous fluid of density ρ and viscosity μ. The outer cylinder is fixed but the inner cylinder rotates with angular speed Ω while translating along its own axis with speed W. The Navier-Stokes and continuity equations are, in cylindrical coordinates,

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^{2}}{r}+v_z\frac{\partial v_r}{\partial z}\right)=$$
$$\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}+\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]-\frac{\partial p}{\partial r}+\rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=$$
$$\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{\partial^2 v_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right]-\frac{1}{r}\frac{\partial p}{\partial \theta}+\rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]-\frac{\partial p}{\partial z}+\rho g_z$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r)+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial z}=0$$

a) Simplify the equations under the assumption that the flow is steady, body forces are absent and there is no applied axial pressure gradient. Describe any additional assumptions that you feel are relevant and simplify accordingly.

b) Nondimensionalize the resulting equations choosing appropriate scales for all variables and pose appropriate boundary conditions.

c) Solve the resulting equations to determine the velocity field.