GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

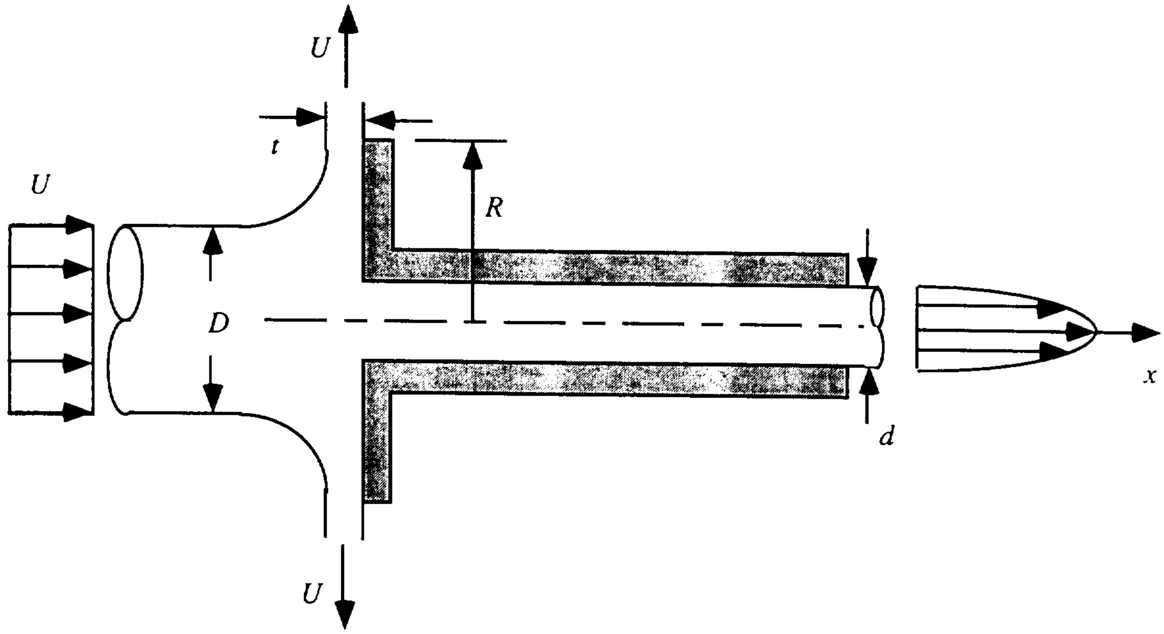
Ph.D. Qualifiers Exam - Fall Semester 1999

Fluid Mechanics
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

WORK ALL PROBLEMS—ALL PROBLEMS ARE OF EQUAL WEIGHT



1. A flange of exterior diameter $2R$ is attached to a tube of inside diameter d. A jet of water of diameter D approaches the flange with uniform speed U. The flange deflects a portion of the jet into an axisymmetric sheet of thickness t at the point where it leaves the flange with negligible change in speed. The remainder of the water passes through the tube, developing into a laminar parabolic velocity profile of the form

$$u(r) = \frac{3}{2}U\left(1 - \frac{4r^2}{d^2}\right),$$

where the maximum speed at the centerline is 3/2 the speed of the approaching flow.

a. If half of the mass of the incoming jet is deflected by the flange, determine both the sheet thickness t and the diameter d necessary to accomplish this.

b. For the case considered in part a, determine the force required to hold the flange and tube in place.

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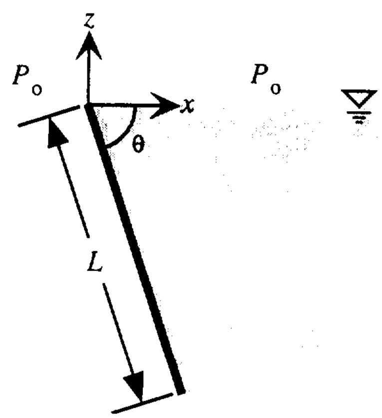
2. Consider a liquid where compressibility effects are significant. The simplest model for a compressible liquid assumes a constant sound speed in the liquid a. Pressure P and density ρ variations are then described by the following relation:

$$dP = a^2 d\rho \, .$$

a. Find the density and pressure distributions, $\rho(z)$ and $P(z)$, respectively, in a compressible liquid as a function of depth z, given $\rho(0) = \rho_o$ and $P(0) = P_o$.

b. Find the net force per unit width due to this pressure distribution on a flat plate of length L and angle of inclination θ immersed in a compressible liquid with its top edge at the free surface ($z = 0$), as shown below. Please give net force in terms of its horizontal (x) and vertical (z) components. Note that there is compressible liquid only on one side of the plate; the pressure on the other side is P_o.



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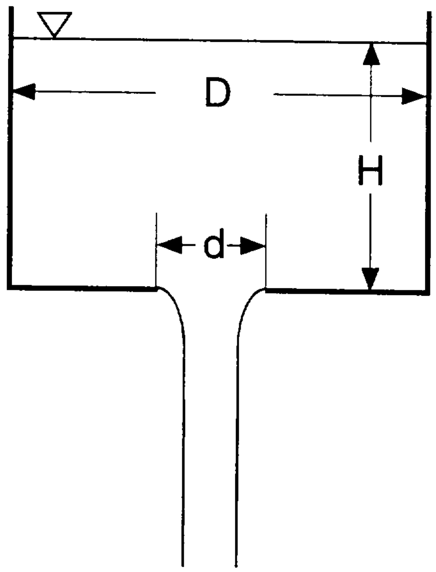
3. The liquid in the cylindrical storage tank shown below is drained through a small round orifice ($d/D \ll 1$) at the bottom. It is desired to design a small-scale model of the system to measure the change in liquid level ΔH.



a. Using dimensional analysis, determine ΔH as a function of the other dimensionless parameters.

b. How does the drain time in the model scale relative to the prototype?

c. If viscous effects can be neglected, can the same fluid be used for the model and prototype? Explain.

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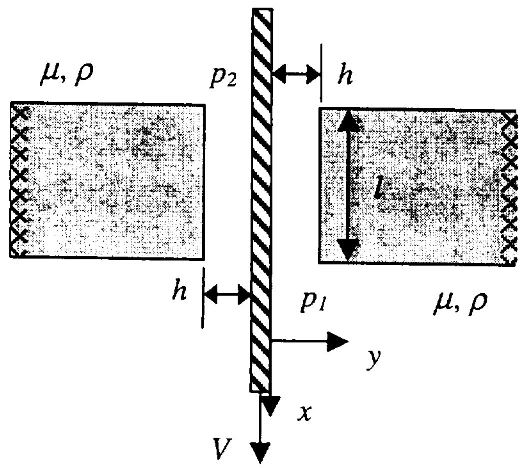

4. A long vertical plate (hatched) is being pulled at a steady velocity V through a slit in a horizontal plate (light gray) as shown in the figure below. An incompressible liquid of density ρ and viscosity μ flows through the gap between the two plates driven by the motion of the vertical plate and a pressure change from the bottom to the top of the horizontal plate $\Delta p = p_1 - p_2$. The gap width h on each side of the vertical plate is much smaller than the length l of the gap. Assuming two-dimensional flow and ignoring gravity, find the following:

a. the magnitude and *sign* of the pressure change along the gap so that the net mass flux through the gap is zero, and

b. the magnitude and *direction* of the force per unit width exerted by the fluid on the vertical plate.

The two-dimensional Navier-Stokes equations and the continuity equation are

$$\rho\left\{\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right\}=-\frac{\partial p}{\partial x}+\rho g_x+\mu\left\{\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right\}$$

$$\rho\left\{\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right\}=-\frac{\partial p}{\partial y}+\rho g_y+\mu\left\{\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right\}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$