GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Quarter 1996

FLUID MECHANICS

EXAM AREA

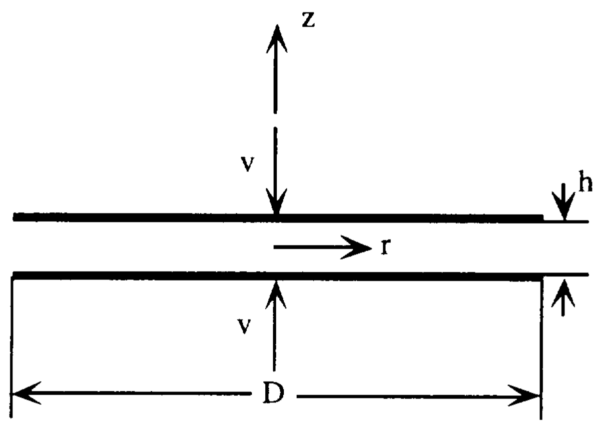
Assigned Number **(DO NOT SIGN YOUR NAME)**

-- **Please sign your name on the back of this page** --

1. Two parallel discs of diameter D are brought together in air along their common axis of symmetry each with a constant speed v, as shown below. When the width of the gap between the discs, h, is small compared to D, it may be assumed that the distribution of radial velocity around the circumference of the discs where $r = D/2$ is uniform.

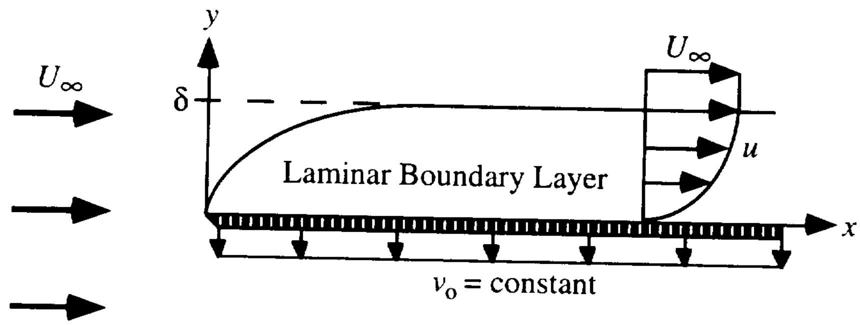
   a. Determine the velocity of the air at $r = D/2$ halfway between the discs (i.e., $z = 0$).

   b. Assuming that the pressure around the perimeter of the discs is atmospheric, can you use Bernoulli's equation to determine the stagnation pressure halfway between the discs at $r = 0$? Explain!



2. Consider a flat plate at zero incidence in a uniform flow of velocity U_∞. A laminar boundary layer forms on this flat plate. Fluid is removed (using suction) from the main flow through a porous surface at a constant velocity v_o ($v_o << U_\infty$) such that the boundary layer achieves, after an initial downstream development distance, an asymptotic, or zero-growth, state.

   a) What are the basic differential equations and associated boundary conditions which describe the velocity field in the asymptotic region of this laminar boundary layer?
   b) Solve these equations to obtain the velocity distribution in this flow.
   c) Finally, what is the friction coefficient at the surface of the plate?



3. Consider an ideal two-dimensional stagnation flow of an incompressible inviscid fluid given by potential function

$$\Phi = 0.5\,A\,(x^2 - y^2)$$

where A is a function of time and is given by $A = A_o + \cos(\omega t)$. The constants, A_o and ω determine the average velocity and the frequency of oscillation.

(a) Determine the x and y components of the velocity field for this flow.

(b) Find the streamfunction and the equation for the streamlines.

(c) Find the equation for the fluid particle pathlines.

(d) At time t=0, fluid particle P_1 is located at (x,y)=(1,1). At times t=1 and t=2, the same fluid particle is found at $(x,y)=(e, e^{-1})$ and $(x,y)=(e, e^{-6})$, respectively. Find the value of ω and A_o for this flow. [Note: $ln(e)=1$]

(e) Draw the fluid particle pathlines including the pathline for particle P_1.

4. A layer of liquid (of depth d) flows steadily over a horizontal smooth plate, as shown above. The liquid surface is open to the atmosphere (free surface), and an obstruction of height h is placed on the plate as shown. The obstruction distorts the flow, so that the height of the free surface above the obstruction is ℓ.

For the prototype flow:

$d_p = 1$ m

$V_p = 5$ m/s

$h_p = 1.2$ m

$\rho_p = 700\ \text{kg/m}^3$

$\mu_p = 3\text{x}10^{-4}\ \text{N-s/m}^2$

We would like to do tests on a 1/10 scale model ($d_m = .1\text{m}, h_m = .12\text{m}$) to find ℓ and the force exerted on the obstruction, by the fluid, F. We will use water in the model test ($\rho_m = 1000\text{kg/m}^3$, $\mu_m = 1.3\text{x}10^{-3}\text{N-s/m}^2$).

a) First assume that viscous effects are negligible. Find the velocity V_m that should be used in the model test.

b) In the test of part a, ℓ_m is found to be .07 m. What would be the corresponding height in the prototype, ℓ_p ?

c) In the test of part a, the force on the obstruction is measured at $F_m = 5\text{N}$. What would be the force in the prototype, F_p ?

Now assume that the viscous effects are much more important than the effects of gravity. If this were the case, what value of velocity V_m should be used in the model test?