Dynamics Systems & Control Ph.D. Qualifying Exam Spring 2017

Instructions:

Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the 3 problems that you select, show all your work in order to receive proper credit. You are allowed to use a calculator.

Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

An electric motor drive system is used to turn a wheel through a set of gears with ratio N_2/N_1 (where N_1 is the number of teeth on the gear on the input shaft, and N_2 is the number of teeth on the gear on the output shaft). The viscous friction on the input shaft is given by b_1 , and the viscous friction on the output shaft is given by b_2 . Let I_1 represent the combined inertia of the motor shaft, input shaft, coupling and first gear. Let I_2 represent the combined inertia of the second gear, output shaft, and load.

Develop a dynamic model that relates the voltage input to the motor (*V*) to the angular velocity of the output shaft (ω_o). **Provide the Laplace domain (transfer function) representation of this model.** The equations that govern the torque, current, and voltage relationships for a DC motor are given below:

$$I = (V - V_{bemf})/R$$

 $V_{bemf} = K_E \omega_i$

 $T = K_T I$

R = terminal resistance of the motor I = current through the motor $K_E = \text{back-EMF constant of the motor}$ $K_T = \text{motor torque constant}$ $V_{bemf} = \text{back-EMF voltage}$ T = torque exerted on motor (input) shaft



Output Shaft

Consider a dynamic system whose input-output transfer function G(s) is given by

$$\frac{Y(s)}{R(s)} = G(s) = \frac{2s+1}{s^2 - 3s + 2}$$

a) What is the definition for a stable system and unstable system?

b) Based on your definition, is the system stable?

c) Suppose $u(t) = \delta(t)$ (Dirac delta function), y(0) = 1, $\dot{y}(0) = 2$. Find y(t), $t \ge 0$.

d) Using the same input, do there exist other initial conditions y(0) and $\dot{y}(0)$ which render $y(t) = 0, t \ge 0$?

e) Following (4), if your answer is yes, find all these initial conditions. Furthermore, does your answer contradict your answer in (a)? If your answer is no, stop.

Consider the following feedback system with a loop gain k (>0). z is a real number.



a) Assume the unity proportional controller or K(s) = 1. Also assume z = -1.

(a-1) Sketch the root-locus plot of the closed-loop system. Determine the angles of departure from complex poles, intersection of asymptotes, break-in/away points, if they exist.

(a-2) Determine the range of *k* such that the closed-loop system is *stable*.

(a-3) Determine k such that the closed-loop system is stable and critically damped.

(a-4) Determine the range of k such that the closed-loop system is *stable* and *underdamped*.

(a-5) You want to determine k such that a typical response of the closed-loop system is oscillatory and its frequency of oscillation is the *greatest*. Specify such closed-loop pole(s) on the root-locus diagram you gave in (a-1). Briefly explain why such pole(s) satisfy the requirement. No calculation is necessary.

b) Assume the unity proportional controller or K(s) = 1.

Find z so that the closed-loop system is always overdamped when the system is stable. The answer is not unique. Justify your answer using the root-locus method. No calculation is necessary.

c) Assume z = 3. A control engineer proposes to use $K(s) = \frac{1}{s-1}$ saying that the controller

cancels the unstable zero at 1 and stabilizes the closed-loop system for any k (>0). Explain why such a controller should NOT be used practically.

Consider a feedback system shown below where $G_p(s) = \frac{1}{s(s+1)(0.5s+1)}$.



a) Sketch the bode diagram of $G_p(s)$. Determine the gain and phase margins.

b) Design a lag compensator of the form $G_c(s) = K \frac{(s/a)+1}{(s/b)+1}$ so that the static velocity error constant (to a unit ramp reference input) is 5 sec⁻¹ without significantly affecting the phase and gain margins.

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