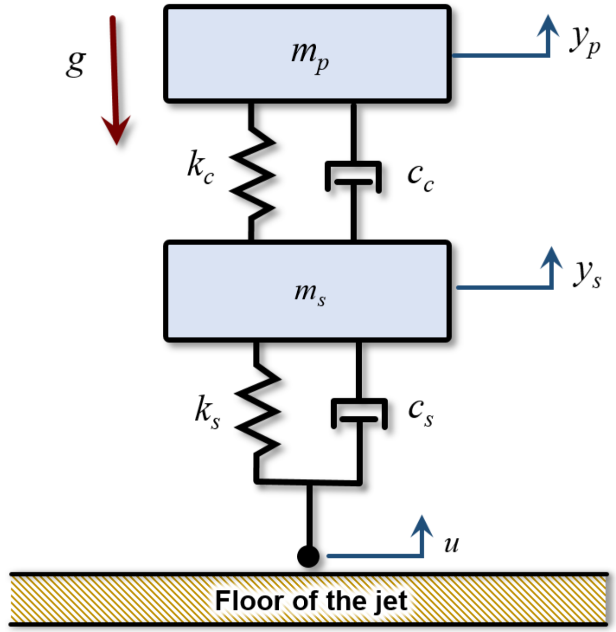
Dynamics Systems & Control Ph.D. Qualifying Exam
Spring 2018

Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the 3 problems that you select, show all your work in order to receive proper credit. You are allowed to use a scientific, non-graphing calculator. Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1: The system illustrated below a model of shock absorbing seat for a hypersonic skip-glide jet. The mass m_p represents the passengers body mass, k_c and c_c represent the stiffness and damping of the seat cushion, mass m_s is the mass of the seat, and k_s and c_s represent the stiffness and damping of the seat suspension mechanism. Input u is the attitude of the jet with respect to the ground. The quantities y_p and y_s are the displacements of the passenger and seat, respectively, when the jet is on the ground (u=0). Gravitational acceleration g is constant.

a) Find the transfer function $Y_p(s)/U_j(s)$ for this system.

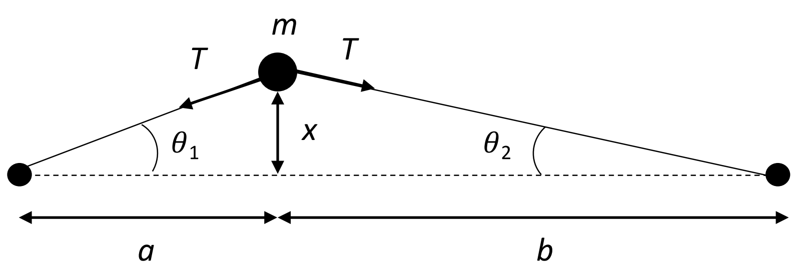


b) Let's say that two people are sitting in a pair of these seats, and that one passenger is twice the mass of the other (e.g. a parent and child). Quantitatively discuss the difference in response characteristics between these two independent systems when subjected to a step input (i.e. sharp jet ascent), assuming both seats have the same mechanical properties.

c) If the jet undergoes sudden turbulence, in the form of a unit impulse, how would those response characteristics change? Would the amplitude of passenger motion be greater during ascent or descent?

d) If you have control parameters k_s and c_s, for the seat suspension, how would change them to reduce the amplitude a passenger's vertical motion (relative to the floor of the jet)?

Problem 2: A mass m is tied to a string which is under a T. The string is attached to walls at two endpoints as shown. Assume that the tension T is constant for any small vertical displacement x of the mass. For the remainder of this problem, neglect gravity, neglect any material damping from the string or drag from the air, and assume that x is small compared to the length of the string (given by $a+b$).



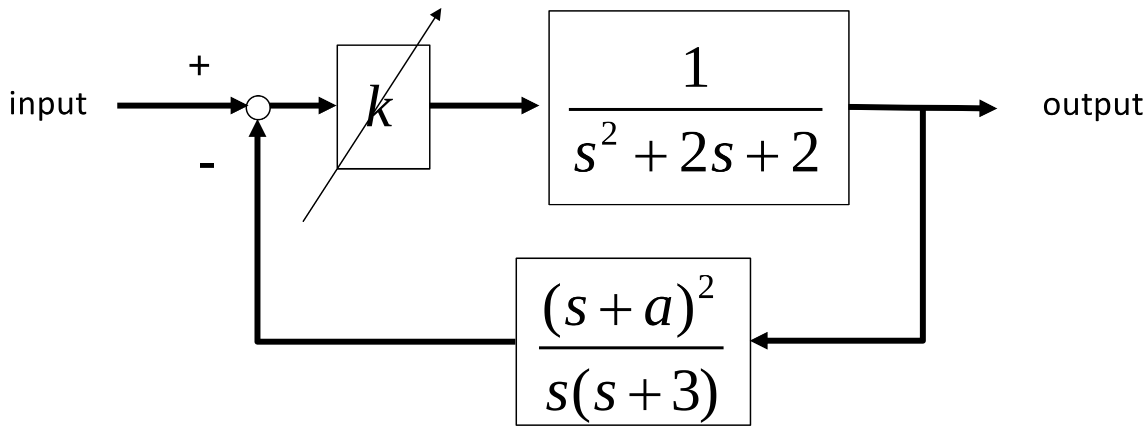
a) Derive the equation of motion for this system.

b) Give a very brief description of the stability properties of this system and the location of the system poles in the complex plane (1-2 sentences should be sufficient).

c) Find an expression for the natural frequency of the system. Suppose the string is 100 cm long, the tension is 1N, and the mass is 1kg. Where should the mass be placed on the string to achieve a natural frequency of 2 rad/s?

d) Suppose that x and $\dot{x}$ can be measured, and we place an actuator in the vertical direction that applies a force F to mass. Design a feedback controller such that the system is critically damped (specify the controller gain(s), assuming the same system parameters as in part c).

Problem 3: Consider the following feedback system with a loop gain k (>0) and a parameter a ($-\infty < a < \infty$).



(a) Consider the case where a=2.

(a-1) Sketch the root-locus plot. Determine intersection of the asymptotes, break-in/away points, if exist. You do not need to determine the angle of departure/arrival from /to complex poles/zeros.

(a-2) Discuss the stability of the closed-loop system with respect to the magnitude of k.

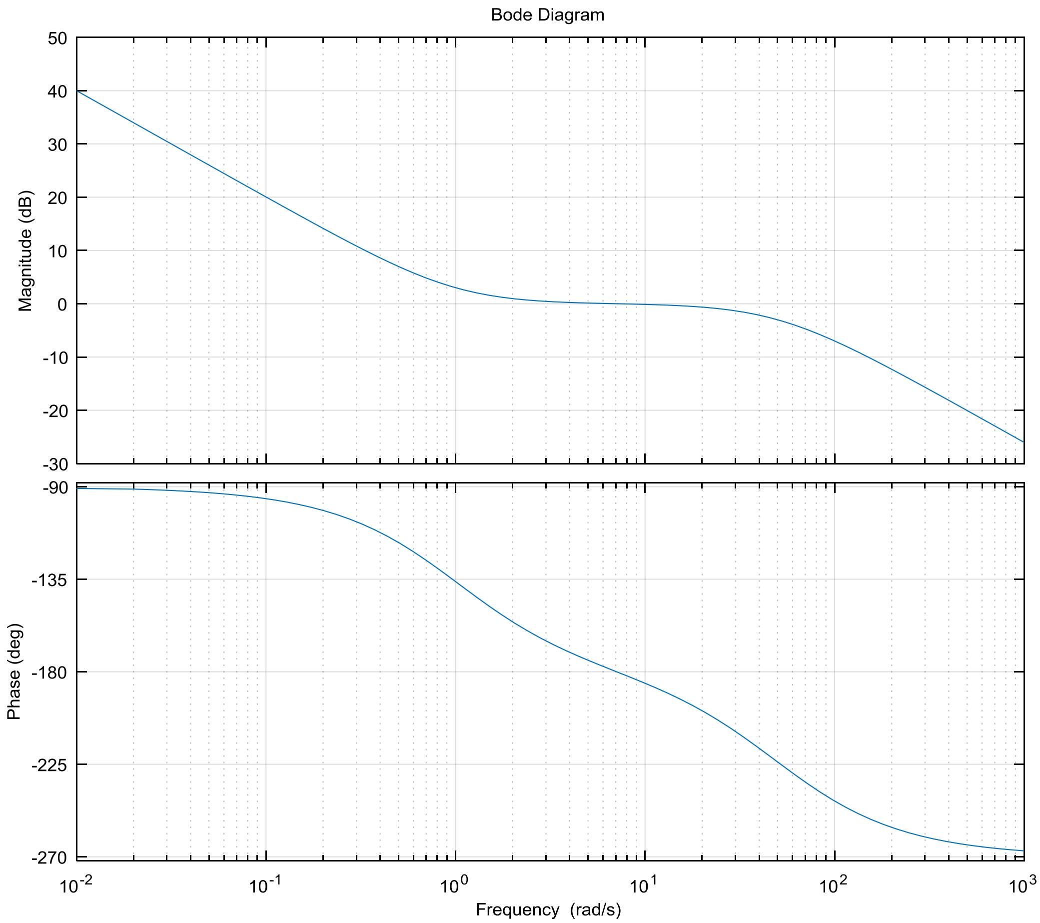
(a-3) There exists a value of k such that the closed-loop system is critically damped. TRUE or FALSE? Briefly explain why.

(b) Find the range of a so that the closed-loop system is stable for ANY k (>0)

(c) Discuss any potential issues regarding the closed-loop stability when a=3.

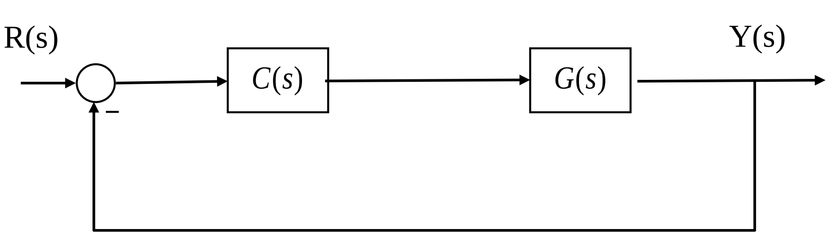
(d) Assume the loop-gain and parameter a are chosen appropriately so that the closed-loop system is stable. Show that the closed-loop system has a steady-state-error for a step input.

Problem 4: The bode diagram of a linear time invariant system with transfer function G(s) is given by



a) Find G(s) from the bode diagram.

b) Design a suitable feedback controller C(s) (e.g., P,PD,PI,PID,Lead,Lag,…) such that the compensated system C(s)G(s) (see the block diagram below) has a phase margin of at least 50 degrees and a gain cross-over frequency (i.e., the frequency at which $|CG(j\omega_c)|=1$) of $\omega_c \geq 10$ rad/sec. Find all the gains/parameters associated with your controller.



c) Sketch the bode diagram of the compensated system C(s)G(s) you found in (b).