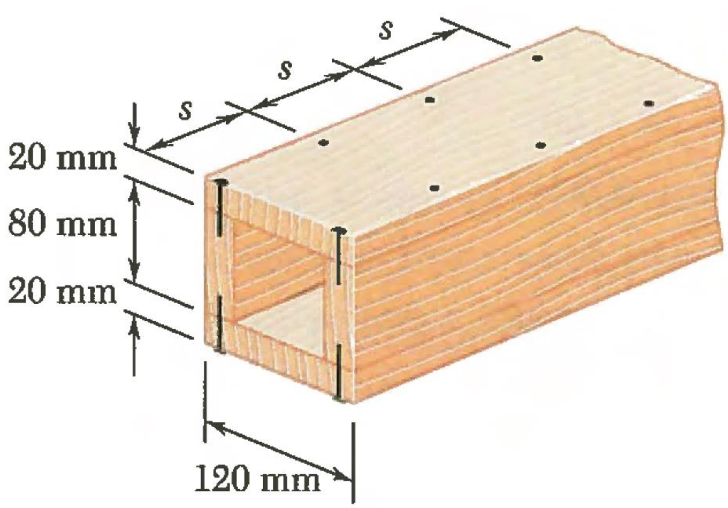
PLEASE NOTE:
You need to correctly answer only 3 out of the 4 problems to receive full credit. In case you attempt all 4 problems, clearly state which 3 problems you want to be graded. If you do not explicitly so indicate, the 3 lowest scores will be used.

Problem #1

A square box beam is made of two 20 x 80 mm planks and two 20 x 120 mm planks nailed together as shown.



(a) If the vertical shear in the beam is $V = 1200$ N, what should the spacing between the nails, *s*, be to ensure that the shearing force in each nail remains below 150 N?

(b) If the interfaces between the planks are also glued as well as nailed, explain how the problem changes. No calculations needed for this part.

(c) If instead the vertical shear acting on the beam is zero, but the bending moment is 500 N-m, what is the shearing force in each nail, assuming the spacing between the nails is the same as determined in part (a)?

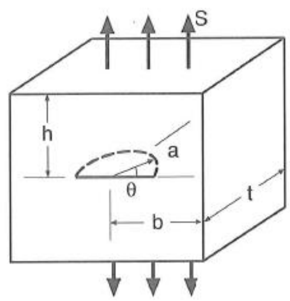
Problem #2

(a) An aluminum alloy (yield stress σ_0 = 300 MPa) has the following fatigue crack propagation relationship for $R = S_{min}/S_{max} = 0$:

$$\frac{da}{dN} = 2.5\times10^{-10}(\Delta K)^4 \text{ m/cycle}$$

with $\Delta K = K_{max} - K_{min} = K_{max}$ having the units of MPa.(m)$^{1/2}$. A component that is made of this material is subjected to constant amplitude, zero-to-maximum loading service. The component is inspected every 36,000 cycles using a technique capable of detecting a crack of 0.5 mm in length on the surface (a_i = 0.25 mm). Assume the crack size at failure to be a_f = 25 mm. Assume that the crack depth is much smaller than the component's thickness and width. Assuming that the surface crack remains semi-circular throughout the life, determine the maximum stress range, $\Delta S = S_{max}$, to get a safety factor in life of 3 (X_N = 3). List all your other assumptions.

Note: For a semicircular surface crack as shown below, we have $K = 0.728S\sqrt{\pi a}$, with 10% accuracy for a/t < 0.4 and a/b < 0.3



(b) The fatigue life N_f of a smooth specimen subjected to a fully reversed cyclic loading between two constant strain values ($R = \varepsilon_{min}/\varepsilon_{max} = -1$) is related to the strain amplitude, ε_a ($\varepsilon_a = \varepsilon_{max}$) by the following equation:

$$\varepsilon_a = \frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c$$

where E, σ'_f, ε'_f, b and c are material constants. On the right side of this equation, the first term corresponds to the elastic strain amplitude (ε_{ea}) while the second term corresponds to the plastic strain amplitude (ε_{pa}).

B1. Approximate N_f as a function of ε_a and the appropriate material constants in the low-cycle fatigue regime. Justify your answer.

B2. Approximate N_f as a function of ε_a and the appropriate material constants in the high-cycle fatigue regime. Justify your answer.

B3. Provide a fatigue life, N_t, as a function of the aforementioned material constants, corresponding to the transition between low- and high-cycle fatigue regimes. Justify your answer.

Problem #3

The ends of a cylindrical rod of cross-sectional area *A* are attached to rigid supports on both sides. The rod is initially stress free, and an axial force of *F* is then applied to a cross-section at distances of *a* and *b* from the two ends (a<b). The rod is made of an elastic-perfectly plastic material, which is characterized by Young's modulus *E* and yield stress σ_s.

(a) Determine the yield load F_y at which the rod starts to yield and the corresponding displacement u_y at point C.
(b) Determine the plastic load F_p that is just sufficient to cause yielding throughout the rod and the corresponding displacement u_p at point C.
(c) If the load is increased to F_p and then removed, find the distribution of residual stress and the residual displacement at point C.

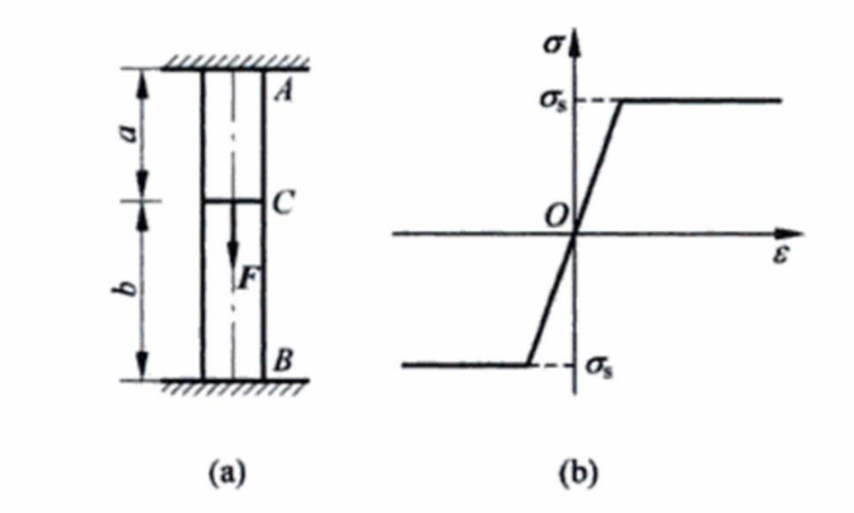


Figure 1

Problem #4

A rigid steel plate is supported by two concrete and one wood posts at their free ends as shown in the figure. The horizontal distances between the wood and concrete posts are equal. The cross-sectional area of all three posts A=40,000 mm^2 and the length L=2 m. Before the load P is applied, the wood post in the middle is shorter than the others by amount *s*=1.0 mm. The Young's moduli for wood and concrete are 25 GPa and 30 GPa, respectively. The allowable compressive stresses for wood and concrete are 10 MPa and 20 MPa, respectively.

(a) Determine the maximum allowable load P_{allow}.
(b) This system is previously in an environment of 25 °C. If the temperature of environment increases to 55 °C, with the linear thermal expansion coefficient of wood and concrete being $22*10^{-6}$/K and $12*10^{-6}$/K, respectively, would the maximum allowable load P_{allow} increase or decrease? (Note: the free ends can move freely when temperature rises). **Show necessary calculations in the answer.**