PhD Qualifying Exam
Heat Transfer, Written Exam

Problem 1

A thin walled spherical tank of radius R_i is used to store liquid nitrogen at T_i. Insulation is wrapped around the tank with a contact resistance per unit area of $R''_{t,c}$. The outside surface of the insulation is exposed to convection environment with heat transfer coefficient of h, and ambient temperature T_f. The outer radius of the insulation is fixed at R_o.

1. (20%) Draw a suitable thermal resistor network, and write an expression for the heat leak rate into the liquid nitrogen, per unit volume.
2. (30%) If R_i can be varied, and the contact resistance is negligible, what value of R_i will give the least volumetric heat transfer rate into the liquid nitrogen ?
3. (20%) Physically explain why a minimum exists for R_i in Part 2.
4. (30%) How will the result in Part 2 change if contact resistance is included.

Problem 2

Consider steady, fully developed, laminar forced convective flow inside a horizontal circular pipe, where the heat transfer coefficient is defined by,

$$q = h(T_s - T_m)$$

where T_s is the pipe's inner surface temperature and T_m is the fluid's mean/average temperature weighted by the velocity. In this problem the pipe wall is hotter than the fluid, and the fluid has a centerline temperature of T_0, where the pipe's radius is R.

1. **[3 points]** Determine the velocity profile in terms of the average velocity u_m
2. **[1 points]** Assuming the temperature profile is known and is equal to:

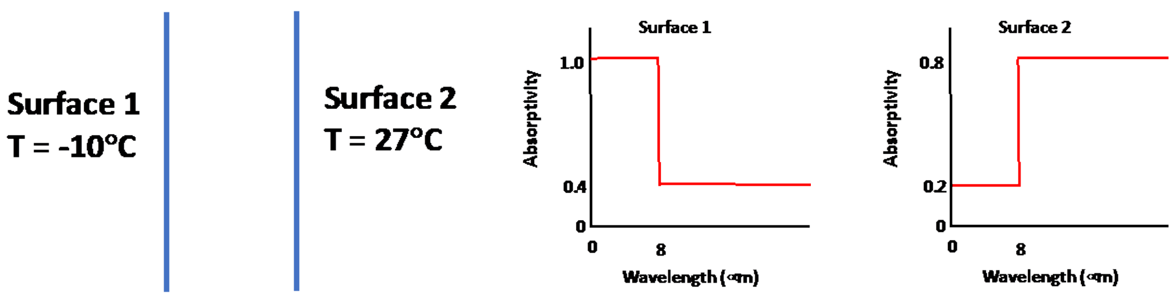
   $$T(r) = T_s + (T_0 - T_s)\left[1 - \left(\frac{r}{R}\right)^2\right]$$

   Write an expression for the average temperature in the fluid T_m, whereby the average is weighted by the velocity profile.
3. **[5 points]** Derive an expression for the heat transfer coefficient h
4. **[1 points]** Derive an expression for the Nusselt number Nu

Problem 3

An industrial freezer is made with vacuum insulated panels that are opaque and have diffuse surface properties shown in the graph below, spectra hemispherical emissivity. The outer wall is maintained at 27°C while the inner wall is maintained at the freezer set point of -10°C. The 5cm gap between the two walls of the panel are evacuated. The height of the wall is 200 cm.

1) **[4 points]** Determine the thermal load provided to the freezer through the vacuum insulated panel. State any assumptions that you make to solve the problem under steady-state conditions with constant properties.
2) **[2 points]** Would the insulation performance improve or decrease if both panels were fabricated with a single material-1, i.e., material of surface-1?
3) **[4 points]** Consider that the vacuum insulated panel leaks over time and becomes filled with air. Determine the heat load to the freezer with the same panel temperatures. Heat transfer between the panels due to conduction can be neglected. What should be done to reduce the heat load to the freezer without adding a radiation shield inside of the vacuum panel (you can change the geometry and surface properties of the vacuum panel). Assume air properties of: $\nu = 13.84 \times 10^{-6}\ m^2/s$, $k = 0.0245\ W/mK$, $\alpha = 19.5 \times 10^{-6}\ m^2/s$, $Pr = 0.71$.



Free Convection in a Cavity:

$Nu = 0.046 Ra^{1/3}$

TABLE 12.1 Blackbody Radiation Functions

λT (μm · K)	$F_{(0 \to \lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ (μm · K · sr)$^{-1}$	$\dfrac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373
3,400	0.361735	0.681544	0.943551
3,600	0.403607	0.650396	0.900429
3,800	0.443382	0.615225×10^{-4}	0.851737
4,000	0.480877	0.578064	0.800291
4,200	0.516014	0.540394	0.748139
4,400	0.548796	0.503253	0.696720
4,600	0.579280	0.467343	0.647004
4,800	0.607559	0.433109	0.599610
5,000	0.633747	0.400813	0.554898
5,200	0.658970	0.370580×10^{-4}	0.513043
5,400	0.680360	0.342445	0.474092
5,600	0.701046	0.316376	0.438002
5,800	0.720158	0.292301	0.404671
6,000	0.737818	0.270121	0.373965
6,200	0.754140	0.249723×10^{-4}	0.345724
6,400	0.769234	0.230985	0.319783
6,600	0.783199	0.213786	0.295973
6,800	0.796129	0.198008	0.274128
7,000	0.808109	0.183534	0.254090
7,200	0.819217	0.170256×10^{-4}	0.235708
7,400	0.829527	0.158073	0.218842
7,600	0.839102	0.146891	0.203360
7,800	0.848005	0.136621	0.189143
8,000	0.856288	0.127185	0.176079
8,500	0.874608	0.106772×10^{-4}	0.147819
9,000	0.890029	0.901463×10^{-5}	0.124801