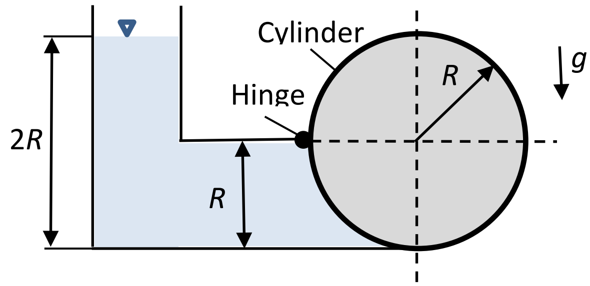
1. A hinged cylindrical gate with the radius R and width b is resting at ground level with one quarter of its circumference submerged in water with specific weight γ. The depth of water is $2R$.



a) Find the magnitude and direction of the hydrostatic force F acting on the cylinder.

b) Determine the minimum weight of the cylinder W that is required to keep the gate closed.

2. Consider pulsatile flow in a circular tube with flow rate, $Q(t) = Q_o \sin \omega t$, where ω is the frequency at 6 rad/s. An experimental model with tube diameter two times larger is to be used to determine the instantaneous pressure difference per unit length, Δp_l, along the tube. Assume that $\Delta p_l = F(D, Q_o, \omega, t, \mu, \rho)$. Here D, t, μ, and ρ are tube diameter, time, dynamic viscosity, and density, respectively.

a) Determine all the dimensionless parameters and the similarity requirements for the model.

b) If the same fluid is used, at what frequency should the model operate.

3. A circular liquid jet (diameter d) of constant density ρ, is moving at a (uniform) constant velocity U (relative to the stationary frame of reference r-z), until it impinges on a circular piston as shown schematically in Figure 1 below. The fluid of the primary impinging jet spreads symmetrically along the surface of the piston and forms a radial secondary jet. It may be assumed that: ***i.*** Viscous effects along the surface are negligible so that the radial velocity of the secondary liquid jet is uniform in azimuthal cross sections normal to the surface; ***ii*** Gravitational and surface tension effects are negligible; and ***iii.*** The pressure near the nozzle and the piston surface is atmospheric.

a) Using an appropriate control volume determine the magnitude and direction of the force F_{ps} that is necessary to hold the piston stationary.

b) Determine the magnitude and direction of the force F_{pm} that should be applied to piston so that it moves to the left or to right with constant velocity $U_p < U$.

c) Assuming that the height of the liquid layer over the surface of the piston (h) at a given radial distance r from the axis of the primary jet remains invariant when the piston is moving, how does the motion of the piston affect the radial velocity of the secondary jet?

d) How does the motion of the piston affect the momentum flux of the secondary jet compared to the momentum flux of the primary jet?

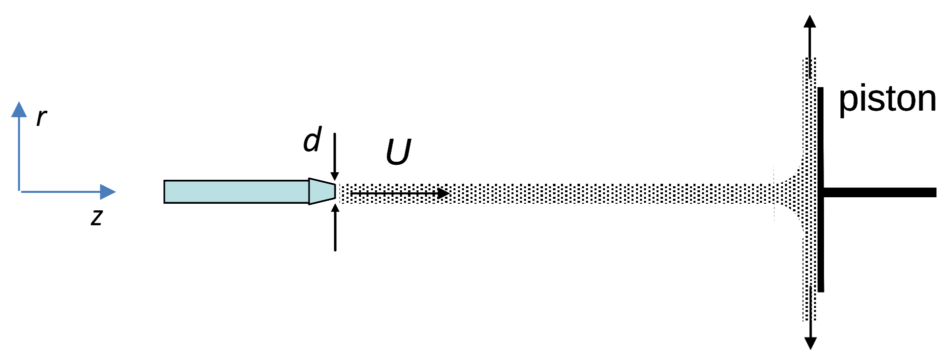
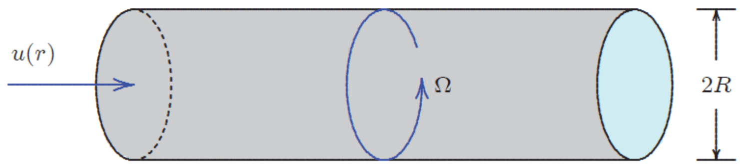


Figure 1

4. Consider fully developed, incompressible, viscous flow in a long pipe of radius R that rotates about its centerline with angular velocity $\mathbf{\Omega}$.



a) Simplify the continuity and Navier-Stokes equations taking advantage of the axial symmetry. State the appropriate boundary conditions.

b) Simplify the equations of Part (a) further for fully developed flow, i.e., solve for the radial velocity, u_r, show that $\partial p/\partial r \neq 0$ and derive ordinary differential equations for the streamwise and circumferential velocities.

c) Solve for the streamwise and circumferential velocities, u and u_θ . *HINT*: Assume a solution for the circumferential velocity of the form $u_\theta = r^m$.

d) Solve for $p(x, r) - p(x, 0)$. Letting x denote a distance along the cylinder axis.

For your reference, the Navier-Stokes equations in cylindrical polar coordinates are:

$$\rho\left(\frac{\partial V_r}{\partial t}+V_r\frac{\partial V_r}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta}-\frac{V_\theta^2}{r}+V_z\frac{\partial V_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\rho g_r$$
$$+\mu\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial(rV_r)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta}+\frac{\partial^2 V_r}{\partial z^2}\right\} \qquad (r)$$

$$\rho\left(\frac{\partial V_\theta}{\partial t}+V_r\frac{\partial V_\theta}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta}+\frac{V_rV_\theta}{r}+V_z\frac{\partial V_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\rho g_\theta$$
$$+\mu\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial(rV_\theta)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial V_r}{\partial \theta}+\frac{\partial^2 V_\theta}{\partial z^2}\right\} \qquad (\theta)$$

$$\rho\left(\frac{\partial V_z}{\partial t}+V_r\frac{\partial V_z}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta}+V_z\frac{\partial V_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\rho g_z$$
$$+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2}+\frac{\partial^2 V_z}{\partial z^2}\right] \qquad (z)$$